New York University
Courant Institute of Mathematical Sciences

G63.2020/G22.2421: Numerical Methods II | *Handout #4*

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Assignment 4

Use double precision computations.

Problem 1

For the ODE $y' = f(t, y)$,
(a) Find the most accurate two step explicit formula (the method should use $y_{n-1},\ y_n,\ y_{n+1}$).
(b) Check the root condition.

Problem 2

For the ODE $y' = f(t, y)$,
(a) Find the most accurate two step implicit formula (the method should use $y_{n-1},\ y_n,\ y_{n+1}$).
(b) Check the root condition. Is the method A-stable?
(c) Use the scheme you constructed to solve the linear system of ODEs

$$\begin{cases} y'(t) = -\begin{bmatrix} 34 & 55 \\ 55 & 89 \end{bmatrix} y, \quad 0 \le t \le 1, \\ y(0) = \begin{bmatrix} \pi \\ 2 \end{bmatrix}, \end{cases} \tag{1}$$

and compute $y(1)$ with at least three digits.
(d) Check the order of convergence numerically.

Problem 3

Exercise 2.3, page 31.

Problem 4

Use the two-step, implicit method constructed in Problem 2 to solve the ODE

$$\begin{cases} y'(t) = \frac{1}{3}\sin(10y) + \cos^2(1+4t), \\ y(0) = 2. \end{cases} \tag{2}$$

(a) Obtain the numerical solution at $t = \pi$ with $h = \pi/100$ and $h/2 = \pi/200$.
(b) Find constants $c_1,\ c_2$ such that $c_1 y_h(\pi) + c_2 y_{h/2}(\pi)$ approximates $y(\pi)$ better than $y_h(\pi)$ and $y_{h/2}(\pi)$ do (this is known as the Richardson extrapolation).
(c) Provide numerical evidence supporting (b).