Assignment 10

Write a (Matlab) code to implement the fast Poisson solver. Note: With matlab, you can use the fast sine transforms, instead of implementing them with the standard forward and backward Fast Fourier transforms. The Matlab implements these linear transforms upto a constant scaling; therefore, if we denote by F and B as the implemented forward and backward (discrete) Fourier transforms, then $F \cdot B = B \cdot F = n \cdot I$. In fact, $1/\sqrt{n} \cdot F$ and $1/\sqrt{n} \cdot B$ are two unitary transforms, one being complex transpose of the other, and one being inverse of the other. The same care must be taken for the sine transforms – you need to scale them properly.

Use your code combined with the fourth order finite difference scheme (see Page 127 of Iserles) to solve the equation

$$\nabla^2 u(x, y) = f(x, y), \quad (x, y) \in D = [0, \pi] \times [0, \pi] \tag{1}$$

subject to the Dirichlet boundary conditions $u(x, y)|_{\partial D} = g(x, y)$.

(a) Check the correctness of your code by choosing special f and g for which you know the exact solution. You need to invent your own way to convince me and youself that the code you write has no bugs.

(b) Once you debug the code, use sufficient number of points to solve the following problem for which $f = 0$, and g is such that

$$\begin{aligned}
u(0, y) &= \sin(ky), \quad y \in [0, \pi], &(2)\\
u(\pi, y) &= \sin(ny), \quad y \in [0, \pi], &(3)\\
u(x, 0) &= 0, \quad x \in [0, \pi], &(4)\\
u(x, \pi) &= 0, \quad x \in [0, \pi]. &(5)
\end{aligned}$$

Make a surface plot for $z = u_h(x, y)$ for $k = 8$, $n = 16$.

(c) Optional. Explain the behavior of the solution.