Assignment 3.

Exercise 1. In the case of Brownian motion, if $\tau$ is a stopping time that takes only a countable number of values show that the process $x(\tau + t) - x(\tau)$ is again a Brownian motion independent of $\mathcal{F}_\tau$.

Exercise 2. If $\tau$ is a stopping time, then show that $\tau_n = \frac{\lceil n\tau \rceil + 1}{n}$ where $\lceil x \rceil$ is the largest integer not exceeding $x$, is again a stopping time. Using the fact that $\tau_n \geq \tau$ and $\tau_n \downarrow \tau$ as $n \to \infty$, extend the strong Markov property for Brownian motion to any stopping time $\tau$ with $P[\tau < \infty] = 1$. 