Problemset 6.

Q1. A function f on a metric space X is uniformly continuous if given any $\epsilon > 0$ tere is a $\delta > 0$ such that $|f(x) - f(y)| < \epsilon$ if $d(x, y) < \delta$. Show that if X is a compact metric space every continuous function is uniformly continuous.

Q2. Give a counter example when X is not compact. Can you make a given continuous function uniformly continuous by changing the metric without changing the topology?

Q3. Show that a metric space X is compact if and only if the Banach space of bounded continuous functions $C(X)$ with the norm $\|f\| = \sup_{x\in X} |f(x)|$ is a separable Banach space (i.e. metric space with the metric $d(f, g) = \|f - g\|$)