Q1. Consider the space $\ell_1$ of sequences $a = \{a_n : n \geq 1\}$ such that $\sum_{n=1}^{\infty} |a_n| = \|a\| < \infty$. The distance between two sequences $a$ and $b$ is defined as

$$d(a, b) = \sum_{n=1}^{\infty} |a_n - b_n|$$

Show that a closed subset $C \subset \ell_1$ is compact if and only if

$$\sup_{a \in C} \|a\| < \infty$$

and

$$\lim_{N \to \infty} \sup_{a \in C} \sum_{n=N}^{\infty} |a_n| = 0$$

Q2. Characterize the compact subsets of the space $\ell^0_\infty$ of sequences $a = \{a_n : n \geq 1\}$ with the property, $\lim_{n \to \infty} a_n = 0$, the metric being

$$d(a, b) = \sup_{n \geq 1} |a_n - b_n|$$