
1. Suppose $\pi_h(x, dy)$ is a Markov Chain transition probability on $R$ such that

$$\lim_{h \to 0} \sup_x \frac{1}{h} \int (y - x) \pi_h(x, dy) - b(x) = 0$$

$$\lim_{h \to 0} \sup_x \frac{1}{h} \int (y - x)^2 \pi_h(x, dy) - a(x) = 0$$

$$\lim_{h \to 0} \sup_x \frac{1}{h} \int |y - x|^3 \pi_h(x, dy) = 0$$

for some smooth functions $a(x), b(x)$ and $u(t, x)$ is a smooth solution of

$$u_t = \frac{1}{2} a(x) u_{xx} + b(x) u_x, u(0, x) = f(x)$$

Then, show that

$$\lim_{h \to 0} \int f(y) \pi_h^n(x, dy) = u(t, x)$$

2. If $u(t, x)$ is a smooth solution of

$$u_t = \frac{1}{2} u_{xx} + b(x) u_x, u(0, x) = f(x)$$

and $v$ is a smooth solution of

$$v_t = \frac{1}{2} v_{xx} + (b(x) + c(x)) v_x, v(0, x) = f(x)$$

show that

$$\sup_x |u(t, x) - v(t, x)| \leq t \sup_x |c(x)| \sup_{t, x} |u_x(t, x)|$$