Homework Set 4. Due March 8.

1. Verify that the function

\[ u(t, x) = \frac{x}{t^{\frac{3}{2}}} e^{-\frac{x^2}{2t}} \]

satisfies the heat equation

\[ u_t = \frac{1}{2} u_{xx} \]

with the initial condition \( u(t, x) \to 0 \) as \( t \to 0 \) for all \( x \). Does this contradict uniqueness? How do you reconcile this?

2. Show that the solution to the heat equation \( u_t = \frac{1}{2} u_{xx} \) with initial condition \( u(0, x) = p(x) \) where \( p \) is a polynomial is given by a polynomial in \( t \) and \( x \). Do explicit calculations for \( p(x) = x, x^2, x^3 \) and \( x^4 \).