Q1. $V$ is a real vector space. Show that the intersection $C = \cap \alpha C_\alpha$ of an arbitrary family of closed convex sets $\{C_\alpha\}$ in $V$, is again a closed convex set in $V$. Show that any closed bounded (compact) convex set is an intersection of half spaces. A half space is a closed convex set of the form

$$\{x : l(x) \leq a\}$$

where $l$ is a linear function on $V$.

Q2. Given a closed convex set $C$, a tangent plane at $x \in C$ is a hyperplane $\{y : l(y) = a\}$, such that $l(x) = a$ and either $l(y) \leq a$ for all $y \in C$ or $l(y) \geq a$ for all $y \in C$. Given a closed convex set $C$, and a linear functional $l$ show that there is at least one point $x \in C$ and a real number $a$ such that $\{y : l(y) = a\}$ is a tangent plane. Show that at every boundary point $x \in \delta C$ there is at least one tangent plane.