Q1. Consider the matrix

\[
A = \begin{pmatrix}
<x_1, x_1> & <x_1, x_2> & \cdots & <x_1, x_n> \\
<x_2, x_1> & <x_2, x_2> & \cdots & <x_2, x_n> \\
\vdots & \vdots & \ddots & \vdots \\
<x_n, x_1> & <x_n, x_2> & \cdots & <x_n, x_n>
\end{pmatrix}
\]

where \( x_1, x_2, \ldots, x_n \) are \( n \) vectors in a complex inner product space. Show that \( A \) is Hermitian and positive semidefinite. It is positive definite if and only if the \( n \) vectors \( x_1, x_2, \ldots, x_n \) are linearly independent.

Q2. \( A \) is a Hermitian and positive definite linear transformation of a complex vector space \( V \to V \). Show that \( \langle \langle x, y \rangle \rangle = < x, Ay > \) defines a new inner product on \( V \). Can you express the new adjoint \( \hat{B} \) of a transformation \( B \) in terms of the old adjoint \( B^* \) and \( A \). When is \( B \) self adjoint with respect to the new inner product?