Homework. Set 6


Q1. If $A$ and $B$ are two linear transformations of $V \to V$ that commute i.e. $AB = BA$, then show that the eigen space

$$W = \{ v : Av = \lambda v \}$$

of $A$ corresponding to any scalar $\lambda$, is an invariant subspace of $B$. In other words

$$BW \subset W$$

Is the same true of the generalized eigen spaces

$$W^k = \{ v : (A - \lambda I)^k v = 0 \}$$

for $k \geq 2$?

Q2. A linear transformation $A$ of $V \to V$ is called semi-simple if for every subspace $W \subset V$ which is invariant, i.e with the property $AW \subset W$ there is a complementary invariant subspace $W'$. Show that every semi-simple linear transformation over the complex numbers can be represented by a diagonal matrix in some basis. Conversely any transformation that is diagonal in some basis is semi-simple.