1. If $T : X \to X$ is $I + F$ where $I$ is identity and $F$ is an operator of finite rank, show that its index is 0 by explicit calculation.

2. Let $H = \{ f : f = \sum_{k=0}^{\infty} a_k e^{ikx} \}$ where $\sum_{k=1}^{\infty} |a_k|^2 < \infty$. $H \subset L_2[S]$ with $S = \{ z : |z| = 1 \}$. $P : L_2(S) \to H$ is the orthogonal projection onto $H$. If $f(s)$ is a continuous function $S \to C$ with $f(s) \neq 0$ for any $s \in S$ then show that $g \to P \frac{1}{f} P f g$ is Fredholm and calculate its index. The map $T_f : H \to H$ is multiplication of $g \in H$, i.e. a function with nonnegative Fourier coefficients by a smooth function $f$ projecting out the negative frequencies, multiplying by $\frac{1}{f}$ and projecting out again the negative frequencies.