Q1. Consider the map

\[(Kf)(x) = \int_{\mathbb{R}^3} \frac{f(y)}{|x - y|} dy\]

acting on \(C_\infty\) functions with compact support on \(\mathbb{R}^3\). Show that \(f\) is \(C_\infty\) and \(\Delta K = cI\) i.e. \(\sum_i \frac{\partial^2}{\partial x_i^2} (Kf) = cf\) for some constant \(c\) and evaluate \(c\).

Q2. What would the corresponding result be in dimension 1 and 2.

Q3. Show that it is not possible for both the function \(f\) and its Fourier transform \(\hat{f}\) to have compact support on \(R\).

Q4. Show that if \(f, \hat{f}\) are Fourier transforms \(\|f\|_2 = \|\hat{f}\|_2 = 1\), then

\[
\left( \int x^2 |f(x)|^2 dx \right) \left( \int y^2 |\hat{f}(y)|^2 dy \right) \geq \frac{1}{4}
\]

Q5. Show that every permutation group has exactly two one dimensional representations, the identity and the signature.

Q6. How many inequivalent representations does the permutation group of 4 elements have and what are their dimensions?