A K3 surface over a field $k$ is a separate, smooth, projective, 2-dimensional scheme $X$ of finite type over $k$ such that its canonical sheaf is equivalent to the structure sheaf $\mathcal{O}_X$ and $H^1(X, \mathcal{O}_X)$ is trivial. K3 surfaces are surfaces of intermediate type, that is, they are neither geometrically rational or ruled, nor of general type. The arithmetic and the geometry of these surfaces have gained a growing interest in the last years, but they are still far from being completely understood.

In this paper, the author presents some of the most recent achievements in topics that are crucial to understand the arithmetic of K3 surfaces.

The paper is composed by five chapters. The basic results about the geometry of K3 surfaces are summarised in the first chapter. The second chapter deals with the Picard lattice of K3 surfaces. In particular it focuses on explicit methods to compute Picard lattices of K3 surfaces. The third chapter is devoted to the study of the Brauer group of a K3 surface. Also in this case, the study focuses on practical methods to compute the Brauer group of K3 surfaces. In the fourth chapter, the author presents an analogy between torsion points on elliptic curves over number fields, and nonconstant Brauer classes of K3 surfaces over number fields. This analogy leads to the formulation of two new conjectures about the number of nonconstant Brauer classes of K3 surfaces over number fields. The fifth and last chapter is about the three project groups that the author organised at the Arizona Winter School 2015. For each project group, the author gives a quick review of the topic of the project and the results achieved by the participants.

The paper is clear and very well written. It presents a variety of results about the arithmetic of K3 surfaces: the core of the paper consists of the presentation of explicit methods to compute the Picard lattice and the Brauer group of K3 surfaces. Most of these results are well known to the experts of the field, except the two conjectures in the fourth chapter, that are original. For each result, a precise reference for the proof is given; for less classical results, a proof is provided in the paper.

The clear, precise, and organic presentation of such a variety of theoretic as well as practical results to study the arithmetic of K3 surfaces, together with two new conjectures, makes this paper a valuable help and source of inspiration for both experts and beginners in this field. Therefore, I strongly recommend its publication.

A list of typos and minor comments follows.

1. Page 1, line -10: at the end of the line, there is a space between $X$ and the coma.
2. Page 3, Example 1.2 and Example 1.4: in these two examples the notion of degree of a K3 surface is used, without having been introduced. Maybe it is worth to introduce it, given that the definition of a K3 surface is given in Definition 1.1, right before Example 1.2.

3. Page 4, line -9: although the notation is somehow standard, the divisors $E_0$ and $E_1$ are not defined.


5. Page 18, line 12: given the expository character of the paper, it might be worth mentioning that the Weil conjectures have been actually proven.

6. Page 23, lines 18, 19: the sentence “These discriminants can be calculated with the Artin-Tate formula (10), and they are, respectively -489 and -5” is misleading. Using the Artin-Tate formula, one can compute the discriminants only up to squares. This sentence could be replaced by one of the following: “The classes modulo squares of these discriminants can be calculated with the Artin-Tate formula (10), and they are, respectively -489 and -5”, or “Using Artin-Tate formula (10), one can show that these discriminants are equivalent modulo squares to -489 and -5, respectively”.


8. Page 32, line -1: in the statement of Proposition 3.14 it looks like there is only one group isomorphism...

9. Page 35, line 12: $x$ is an integer here, not a variable. I would change the line “[...] i.e., for an integer $x$ such that $x^2 \equiv 13 \pmod{16}$ has solution. No such solution exists” with “[...] i.e., for an integer $x$ such that $x^2 \equiv 13 \pmod{16}$. No such integer exists”.

10. Page 39, lines -12, -11: the sentence “We start by exploring this idea, and we argue the analogy suggests it is conceivable that [...]” is unnecessarily convoluted and possibly grammatically wrong. I would change it with “We start by exploring this idea: the analogy suggests it is conceivable that [...]”.

11. Page 41, line -7: two periods inserted after $U^3 \oplus E_1(-1)^2$.

12. Page 42, line 17: there is a typo in the name of one of the authors, the correct name is Nicholls.

13. Page 42, line -3: given that references are used as actual substantives throughout the whole paper, I would put [CFTTV16] in between parentheses.