Entropy Budget of an Atmosphere in Radiative–Convective Equilibrium. Part I: Maximum Work and Frictional Dissipation

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ABSTRACT

The entropy budget of an atmosphere in radiative–convective equilibrium is analyzed here. The differential heating of the atmosphere, resulting from surface heat fluxes and tropospheric radiative cooling, corresponds to a net entropy sink. In statistical equilibrium, this entropy sink is balanced by the entropy production due to various irreversible processes such as frictional dissipation, diffusion of heat, diffusion of water vapor, and irreversible phase changes. Determining the relative contribution of each individual irreversible process to the entropy budget can provide important information on the behavior of convection.

The entropy budget of numerical simulations with a cloud ensemble model is discussed. In these simulations, it is found that the dominant irreversible entropy source is associated with irreversible phase changes and diffusion of water vapor. In addition, a large fraction of the frictional dissipation results from falling precipitation, and turbulent dissipation accounts for only a small fraction of the entropy production.

This behavior is directly related to the fact that the convective heat transport is mostly due to the latent heat transport. In such cases, moist convection acts more as an atmospheric dehumidifier than as a heat engine. The amount of work available to accelerate convective updrafts and downdrafts is much smaller than predicted by studies that assume that moist convection behaves mostly as a perfect heat engine.

1. Introduction

Carnot (1824) recognized that the generation of convective motion by clouds is similar to the production of mechanical work by a steam engine. Since then, the second law of thermodynamics has been applied to various problems within the atmospheric sciences, such as the general circulation of the atmosphere (Peixoto et al. 1991; Peixoto and Oort 1992; Goody 2000), radiative heat transfer (Li et al. 1994; Li and Chýlek 1994), hurricane dynamics (Emanuel 1986; Bister and Emanuel 1998), dust devils (Rennó 1998), and moist convection (Rennó and Ingersoll 1996, henceforth RI; Emanuel and Bister 1996, henceforth EB).

Clausius' formulation of the second law can be written as

$$\Delta S = \frac{Q}{T} + \Delta S_{irr} \tag{1}$$

$$\Delta S_{irr} \geq 0. \tag{2}$$

Here, ΔS is the entropy change associated with a physical transformation, Q is the external heating, T is the temperature of the system, and ΔS_{irr} is the irreversible entropy production. Entropy is a state function of the system: ΔS depends only on the initial and final states. The irreversible nature of physical transformations is imposed by requiring the irreversible entropy production to be positive. Equation (1) is referred to as the *entropy budget*, which consists of analyzing the various processes occurring in a given system and determining their effect on the entropy of the system.

The second law allows one to quantify the irreversibility associated with physical transformations in terms of their irreversible entropy production. We focus here on the four mechanisms responsible for the bulk of the irreversible entropy production in moist convection: frictional dissipation, evaporation of water vapor, diffusion of heat, and diffusion of water vapor.

Among the different sources of irreversibility, frictional dissipation should receive special attention. Be-

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cause of the continuous loss of mechanical energy due to frictional dissipation, mechanical work must be continuously produced to maintain the atmospheric circulation. Carnot's heat engine analogy provides a helpful paradigm: the work required to maintain the atmospheric flow is related to the atmospheric heat transport from the warm parts of the globe to the colder regions. By determining the contribution of frictional dissipation in the entropy budget, one can obtain important information on the behavior of convection. This is the approach followed in RI and EB, who attempt to determine the work performed by convective systems from an analysis of the entropy budget, and use this estimate to construct a theory for the vertical velocity, convective available potential energy (CAPE) and intermittency of moist convection. In both RI and EB, moist convection is assumed to behave mostly as a perfect heat engine where mechanical work is dissipated through a turbulent energy cascade acting on convective updrafts and downdrafts.

A first difficulty with this theory has been discussed by Pauluis et al. (2000, henceforth PBH; see also Rennó 2001; Pauluis et al. 2001), who show that a significant amount of frictional dissipation occurs in the shear zones surrounding falling hydrometeors. In numerical simulations, this precipitation-induced dissipation is found to be larger than the dissipation associated with the turbulent cascade from the convective scales to smaller scales. Most of the work performed by convection is used in lifting water rather than in accelerating the updrafts and downdrafts. However, PBH also observe that the total work performed by the atmosphere is significantly smaller than that expected for a Carnot cycle.

This leads us to ask whether or not moist convection behaves as a perfect engine. We do not question here the fact that the production of mechanical work in the atmosphere is associated with a heat transport from a warm source to a cold sink. However, moist convection does not act solely as a heat engine: it also plays an essential role as an *atmospheric dehumidifier*. The ascent of moist air in deep convective towers results in a removal of water vapor through condensation and precipitation. In radiative–convective equilibrium, this dehumidification is balanced by a moistening of dry air associated with various processes such as surface evaporation, entrainment of tropospheric air into the planetary boundary layer, mixing of clouds into the environment, and reevaporation of rain and snow. This moistening is inherently an irreversible process, associated at the microphysical scale with irreversible phase changes and diffusion of water vapor, and therefore results in a net production of entropy. This important aspect of moist convection is discussed in greater detail in a companion paper (Pauluis and Held 2002, henceforth PH).

These two aspects of convection, acting both as a heat engine and as an atmospheric dehumidifier, are in competition with each other in terms of irreversible entropy production. To the extent that convection behaves as an atmospheric dehumidifier, its ability to function as a heat engine is reduced.

We use a cloud ensemble model (CEM) to simulate radiative–convective equilibrium and analyze the entropy budget. Although these models were initially developed for case studies of convection (Klemp and Wilhelmson 1978; Lipps and Hemler 1982), the increase in computer power makes it possible to simulate large enough domains over a long enough time to achieve radiative–convective equilibrium. Many recent studies have used CEMs to investigate statistical properties of convection (Tao et al. 1987, 1999; Held et al. 1993; Xu and Randall 1999; Shutts and Gray 1999; Tompkins and Craig 1998; Tompkins 2000). Notice that Shutts and Gray (1999) also find some difficulties with the theories proposed in RI and EB, although they do not discuss the entropy budget of the simulations. These models usually treat water through bulk parameterizations which distinguish between water vapor, cloud water, and precipitation, and provide enough information on thermodynamic processes for an analysis of the entropy budget.

Section 2 discusses general aspects of the entropy budget of an atmosphere in radiative–convective equilibrium. In particular, we emphasize how the entropy budget relates to mechanical work, and how the latter is reduced in the presence of other irreversible processes. A fundamental difference between dry and moist convection is that irreversible phase changes and diffusion of water vapor decrease the amount of work performed by the atmosphere.

Section 3 analyzes the entropy budget of numerical simulations of radiative–convective equilibrium with a CEM. Radiative–convective equilibria in dry and moist atmospheres are compared. It is shown that, in the moist case, irreversible phase changes and diffusion of water vapor are the dominant entropy sources. It is also found that frictional dissipation associated with precipitation is larger than the dissipation resulting from a turbulent cascade to small scales, consistent with the findings of PBH. The differences between the entropy budget of the physical atmosphere and that of the numerical model are discussed in the appendix.

Section 4 presents a nondimensional analysis of the entropy budget. This aims at determining a convective efficiency, which is similar to the notion used by Craig (1996). It is shown that this convective efficiency depends on two aspects of moist convection: the relative magnitude of latent and sensible heat transport, and the relative magnitude of the frictional dissipation due to precipitation as compared to the turbulent dissipation of kinetic energy. The theories of RI and EB correspond to the specific case of weak latent heat transport by convection, but significantly overestimate the convective efficiency when the latent heat transport is large. This analysis shows that the entropy budget of our nu-

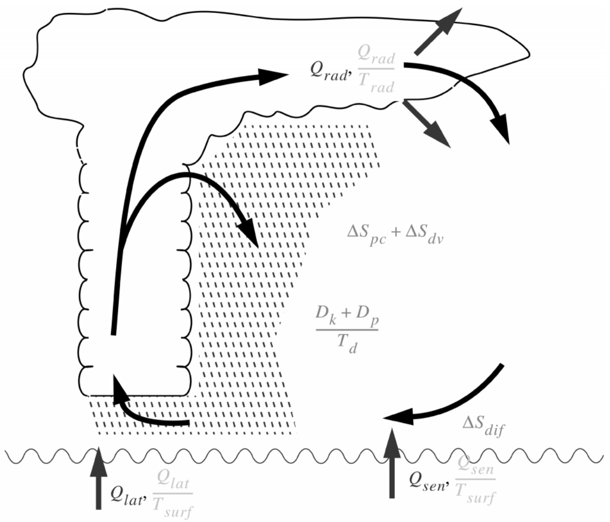

FIG. 1. Schematic representation of the energy and entropy budgets of an atmosphere in radiative–convective equilibrium. The heat sources and sinks are radiative cooling Q_{rad}, surface sensible heat flux Q_{sen}, and surface latent heat flux Q_{lat}. The irreversible entropy sources are frictional dissipation D/T_d, diffusion of heat ΔS_{dif}, diffusion of water vapor ΔS_{dv}, and irreversible phase changes ΔS_{pc}.

merical simulations is illustrative of that of an atmosphere where the convective heat transport is dominated by the transport of latent heat.

The last section discusses the implications of our analysis of the entropy budget for the behavior of moist convection.

2. Entropy budget and frictional dissipation

Consider an atmosphere in radiative–convective equilibrium as described schematically in Fig. 1. The radiative cooling of the troposphere is balanced by surface heat flux, which is decomposed into the sensible heat flux and latent heat flux. This differential heating destabilizes the air column so that convection can develop. There is no large-scale circulation in the sense that there is no mass transport through the lateral boundaries of the system. After some time, the system reaches a statistical equilibrium where differential heating is balanced by convective heat transport. At this point, the total internal energy, mechanical energy and entropy of the atmosphere are statistically steady. Conservation of energy requires that surface fluxes balance radiative cooling:

$$Q_{\rm rad} + Q_{\rm lat} + Q_{\rm sen} = 0. \tag{3}$$

Here, $Q_{\rm sen}$ and $Q_{\rm lat}$ are the surface sensible heat flux and the surface latent heat flux respectively, $Q_{\rm rad} = \int_\Omega f_{\rm rad}$ is the total radiative cooling of the troposphere, $f_{\rm rad}$ is the radiative cooling rate per unit volume, and $\int_\Omega = \int dx\, dy\, dz$ denotes the integral over the entire atmospheric domain.

The mechanical work W done by convection is due to air expansion and can be written as

$$W = \int_\Omega p \partial_i V_i. \tag{4}$$

Here, p is the total pressure, V_i is the ith component of the velocity, $\partial_i = \partial/\partial x_i$ is the partial derivative in the i direction, and the convention of summing over repeated indices is adopted.

Mechanical energy is also removed and converted into internal energy through frictional dissipation. The total dissipation per unit area D is given by

$$D = D_k + D_p = \int_\Omega \sigma_{ij} \partial_j V_i, \tag{5}$$

where σ_{ij} is the viscous stress tensor. For precipitating convection, frictional dissipation can be decomposed into the precipitation-induced dissipation D_p occurring in the microscopic shear zones surrounding hydrometeors, and the dissipation D_k associated with the turbulent energy cascade from the convective scales of motion to the smaller scales at which viscosity can act. As discussed in PBH and EB, the precipitation-induced dissipation can be estimated from

$$D_p = \int_\Omega g\rho_c V_T = \int_\Omega \rho q_t g w, \tag{6}$$

where g is the gravitational acceleration, ρ is the mass of air per unit volume, ρ_c is the mass of falling hydrometeors per unit volume, q_t is mass of total water per unit mass of moist air, V_T is the terminal velocity of the falling hydrometeors, and w is the vertical velocity of the air. Equation (6) indicates that D_p is also equal to the geopotential energy imparted to water by the atmospheric flow.

For the system considered here, one can neglect both the work performed by the atmosphere on its lower boundary (e.g., by generating surface gravity waves) and the kinetic energy flux at the same boundary (due to a viscous transfer of kinetic energy to the oceans). In this case, conservation of mechanical energy requires the mechanical work to be dissipated by friction:

$$W - D_p - D_k = 0. \tag{7}$$

The entropy per unit mass of moist air is defined by

$$s = (1 - q_t)(C_{pd} \ln T - R_d \ln p_d) + q_t C_l \ln T + \frac{q_v L_v}{T} - q_v R_v \ln \mathcal{H}. \tag{8}$$

In this expression, q_v is the specific humidity for water vapor, C_{pd} is the specific heat at constant pressure of dry air, C_l is the specific heat of liquid water, T is the temperature of moist air, R_d and R_v are the gas constants of dry air and water vapor, p_d is the partial pressure of dry air, $\mathcal{H} = e/e_s$ is the relative humidity, e is the water vapor pressure, and e_s is the saturation vapor pressure. Expression (8) assumes that moist air behaves as a ideal mixture of ideal gases. In this respect, it is only an approximation of the entropy of moist air. For further discussion, the reader can refer to Emanuel (1994) or Iribarne and Godson (1981).

Entropy changes are due to either a heat exchange with the environment or to an irreversible process, the latter always resulting in a net production of entropy, as required by the second law of thermodynamics. In this study, the external heat sources are limited to radiative cooling and heat exchange at the earth's surface.

The entropy source or sink associated with external heating or cooling is given by the energy input divided by the temperature at which it occurs. Hence, the surface latent and sensible heat fluxes are associated with an entropy source equal to $Q_{\rm lat}/T_{\rm surf}$ and $Q_{\rm sen}/T_{\rm surf}$, where $T_{\rm surf}$ is the surface temperature. Radiative cooling is associated with an entropy sink $Q_{\rm rad}/T_{\rm rad}$ where $T_{\rm rad}$ is an effective cooling temperature defined by

$$\frac{Q_{\rm rad}}{T_{\rm rad}} = \int_\Omega \frac{f_{\rm rad}}{T}. \qquad (9)$$

In statistical equilibrium, various entropy sources and sinks compensate each other. The entropy budget takes the form

$$\frac{Q_{\rm lat} + Q_{\rm sen}}{T_{\rm surf}} + \frac{Q_{\rm rad}}{T_{\rm rad}} + \Delta S_{\rm irr} = 0, \qquad (10)$$

where $\Delta S_{\rm irr}$ is the total entropy production by the irreversible processes. As the second law of thermodynamics requires irreversible processes to be an entropy source $\Delta S_{\rm irr} \geq 0$, the differential heating of the atmosphere must result in a net entropy sink:

$$\frac{Q_{\rm lat} + Q_{\rm sen}}{T_{\rm surf}} + \frac{Q_{\rm rad}}{T_{\rm rad}} \leq 0. \qquad (11)$$

Because of the energy balance $Q_{\rm lat} + Q_{\rm sen} + Q_{\rm rad} = 0$, statistical equilibrium can only be achieved when the effective cooling temperature $T_{\rm rad}$ is lower than the temperature of the heat source $T_{\rm surf}$, as discussed, for example, by Lorenz (1967).

For a given distribution of heat sources and sinks, the entropy budget (10) provides a constraint on the total production of entropy by irreversible processes. By estimating the contribution of individual processes, one can then obtain specific information about convective activity. The contribution of frictional dissipation is of particular interest as it is related to the work performed by the system and is potentially related to fundamental characteristics of convective systems such as CAPE or vertical velocity. The entropy production due to frictional heating ΔS_d is given by

$$\Delta S_d = \int_\Omega \frac{\sigma_{ij}\partial_j V_i}{T} = \frac{D_k + D_p}{T_d} = \frac{W}{T_d}, \qquad (12)$$

where T_d is the effective temperature at which frictional dissipation occurs.

Combining the expressions (3), (7), (10), and (12) yields an estimate of the total mechanical work done by convection:

$$W = \frac{T_d(T_{\rm surf} - T_{\rm rad})}{T_{\rm surf}T_{\rm rad}}|Q_{\rm rad}| - T_d\Delta S_{\rm nf}, \qquad (13)$$

where $\Delta S_{\rm nf} = \Delta S_{\rm irr} - \Delta S_d$ is the irreversible entropy source due to mechanisms other than friction.

The maximum work $W_{\rm max}$ that can be produced by the system for a given distribution of heat sources and sinks occurs when no other irreversible entropy sources are present $\Delta S_{\rm nf} = 0$:

$$W_{\rm max} = \frac{T_d(T_{\rm surf} - T_{\rm rad})}{T_{\rm surf}T_{\rm rad}}|Q_{\rm rad}|. \qquad (14)$$

The difference between the mechanical work in the atmosphere and this theoretical maximum is due to the production of entropy by the other irreversible processes:

$$W_{\rm max} - W = T_d\Delta S_{\rm nf}. \qquad (15)$$

Hence, one needs to estimate $\Delta S_{\rm nf}$ to determine the mechanical work produced by the system. Conversely, it is possible to derive the entropy produced by other irreversible processes from the total mechanical work and the distribution of heat sources and sinks. In the present paper, we focus on four irreversible processes: frictional heating, diffusion of heat, diffusion of water vapor and irreversible phase changes.[1]

It is first argued that the entropy production due to molecular diffusion of sensible heat $\Delta S_{\rm dif}$ is negligible in all cases of interest. The entropy production due to the diffusion of sensible heat is given by

$$\Delta S_{\rm dif} = \int_\Omega -\frac{J_{{\rm sen},i}\partial_i T}{T^2}, \qquad (16)$$

where $J_{{\rm sen},i}$ is the molecular flux of sensible heat in the i direction. In the surface layer, molecular diffusion transports the sensible heat flux $Q_{\rm sen}$ from the earth's surface at $T_{\rm surf}$ into the atmosphere at $T_{\rm surf} - \Delta T_{\rm bnd}$. The entropy production associated with molecular diffusion near the surface can be approximated by

$$\Delta S_{\rm dif,sfc} \approx Q_{\rm sen}\frac{\Delta T_{\rm bnd}}{T_{\rm surf}^2}. \qquad (17)$$

Molecular diffusion of heat occurs also as an end result of turbulent mixing in the atmosphere. A mixing re-

[1] When radiation is treated as part of the system, absorption of shortwave radiation at the earth's surface is the largest irreversible entropy source in the climate system [see Li et al. (1994) and Li and Chýlek (1994) for a discussion of the irreversible entropy production by radiative transfer]. By treating radiative processes as external heat sources and sinks, the entropy production due to radiative transfer and absorption is included in the entropy sources and sinks due to the external heating.

sulting from a heat transfer Q_{mix} from an air mass at temperature $T + \Delta T_{dt}$ to another air mass at temperature T results in an entropy production given by

$$\Delta S_{dif,int} \approx Q_{mix}\frac{\Delta T_{dt}}{T^2}. \qquad (18)$$

For dry convection, the radiative cooling in the environment is balanced by the sensible heat flux from the updraft: $Q_{mix} \approx -Q_{rad}$. Therefore, as long as ΔT_{dt} and ΔT_{bnd} are small in comparison to $T_{surf} - T_{rad}$, the entropy production by molecular diffusion of temperature can be neglected in the entropy budget. Hence, for dry radiative–convective equilibrium, where the only irreversible processes are diffusion of heat and frictional heating, frictional heating accounts for most of the irreversible entropy production. One can then assume that the total work done by dry convection is close to the theoretical maximum: $W \approx W_{max}$.

For moist convection, the surface sensible heat flux is much smaller than the net radiative cooling $Q_{sen} \ll |Q_{rad}|$, and the temperature jump at the lower boundary is small $\Delta T_{bnd} \ll T_{surf} - T_{rad}$. Hence, the entropy production associated with the surface sensible heat flux is negligible in comparison to the total irreversible entropy production. Similarly, the heat transferred at the detrainment level can be written as $Q_{mix} \approx M_{up}C_p\Delta T_{dt}$, where M_{up} is the upward mass transport by convection. The radiative cooling in the free troposphere is approximately balanced by the warming due to subsidence $Q_{rad} + M_{up}\Delta G = 0$, where ΔG is the dry static energy difference between the detrainment level and the subcloud layer. Hence, the diffusion of sensible heat at the detrainment level is $Q_{mix} \approx -Q_{rad}C_p\Delta T_{dt}/\Delta G$. As the vertical difference of dry static energy is much larger than the enthalpy difference between the updrafts and the environment $\Delta G \gg C_p\Delta T_{dt}$, the sensible heat flux associated with detrainment is significantly smaller than radiative cooling $Q_{mix} \ll -Q_{rad}$. Also, the temperature difference between detraining air and the environment is small in comparison to the difference between the surface temperature and the average temperature at which the atmosphere is cooled radiatively, $\Delta T_{dt} \ll T_{surf} - T_{rad}$. Equations (17) and (18) indicate that the entropy production due to molecular diffusion of sensible heat can be neglected in comparison to the other irreversible entropy sources.

In moist convection, there are two additional irreversible processes directly involving water vapor: irreversible phase changes and diffusion of water vapor.

Irreversible phase changes occur when liquid water evaporates in unsaturated air or when water vapor condenses in supersaturated air. (Freezing and melting can also result in an irreversible entropy production, but are not treated explicitly in this paper.) The irreversible entropy production due to the condensation of M kilograms of water is given by $MR_v \ln\mathcal{H}$. The total entropy production due to irreversible phase changes ΔS_{pc} is given by the integral

$$\Delta S_{pc} = \int_\Omega (C - E)R_v \ln\mathcal{H} - \int_{z=0} J_{v,z}R_v \ln\mathcal{H}. \qquad (19)$$

Here, C and E are the condensation and evaporation rates per unit volume, and $J_{v,z}$ is the vertical component of the molecular flux of water vapor. The first term on the right-hand side of (19) is the production due to irreversible condensation and reevaporation in the atmosphere. The second term is the production due to irreversible evaporation at the surface.

The irreversible entropy production by molecular diffusion of water vapor is given by

$$\Delta S_{dv} = \int_\Omega R_v J_{v,i}\partial_i \ln e, \qquad (20)$$

where $J_{v,i}$ is the ith component of the molecular flux of water vapor.

The distinction between production of entropy by diffusion of water vapor and by phase changes is rather artificial: the entropy production associated with evaporation at relative humidity $\mathcal{H} = e_0/e_s$ is the same as would result from diffusion of water vapor from a saturated region with $e = e_s$ to a region where the water vapor is $e = e_0$. In a numerical model, the partial pressure of water vapor represents the average pressure over a grid box and is not representative of the partial pressure in the vicinity of a water droplet. Hence, in a model, the distinction between the contribution of phase changes and diffusion is arbitrary and depends on specific assumptions about the microphysics. Indeed, if it is assumed that all air is saturated in the microscopic vicinity of all condensate surfaces, then phase changes are reversible and diffusion of water vapor is the only irreversibility. However, the total entropy production due to phase changes and diffusion of water vapor is independent of these assumptions, and is treated here as a single entropy source, referred to as irreversible entropy production due to moist processes.

Expressions (19) and (20) require the knowledge of the molecular flux of water vapor and of the relative humidity. However, PH show that the irreversible entropy production by moist processes can also be determined from the convective transport of latent heat:

$$\Delta S_{dv} + \Delta S_{pc} = T_{vap}^{-1}\left(Q_{lat}\frac{T_{surf} - T_{lat}}{T_{surf}T_{lat}} - W_{vap}\right). \qquad (21)$$

Here, W_{vap} is the expansion work by water vapor given by

$$W_{vap} = \int_\Omega e\partial_i V_i = -\int_\Omega (\partial_t e + V_i\partial_i e). \qquad (22)$$

The effective temperature of water vapor pressure changes T_{vap} is obtained from the relationship

$$\frac{W_{vap}}{T_{vap}} = -\int_\Omega \frac{\partial_t e + V_i\partial_i e}{T}, \qquad (23)$$

and the effective temperature of latent heat release T_{lat} is given by

$$\frac{Q_{lat}}{T_{lat}} = \int_{\Omega} \frac{L_{v0}(C - E)}{T}. \qquad (24)$$

Here, L_{v0} is the latent heat of vaporization at surface temperature, so that the latent heat flux at the surface is $Q_{lat} = \int_{\Omega} L_{v0}(C - E)$. We refer the reader to PH for more discussion on the relationship (21) and the various quantities introduced in this equation. The estimates of the entropy production by moist processes in the numerical simulations of section 3 and scalings arguments of section 4 are based on (21).

For moist convection, the irreversible entropy production is the sum of the production due to frictional heating, diffusion of heat, and the moistening effect. Diffusion of heat is small, as seen before. Therefore, we have

$$\Delta S_{irr} \approx \frac{D}{T_d} + \Delta S_{dv} + \Delta S_{pc}. \qquad (25)$$

Hence, the main question is how the irreversible entropy production is split between frictional heating and moist processes, or, from a microscopic perspective, between diffusion of water vapor and diffusion of momentum.

3. Numerical simulations

A cloud ensemble model is used to simulate radiative–convective equilibrium and to analyze the entropy budget and kinetic energy dissipation. The model has been developed by Lipps and Hemler (1982, 1986, 1988) and is used in a version similar to the one used by Held et al. (1993), but with higher horizontal and vertical resolution. Model dynamics is elastic and allows the zonal-mean state of the system to evolve freely in response to convection, and includes a semi-implicit scheme for vertically propagating sound waves. It incorporates a bulk microphysics and separates water vapor, cloud water, rain, and snow. Surface sensible and latent heat fluxes are obtained from a bulk parameterization assuming an ocean surface at a constant temperature of 298 K, with the drag coefficient based on Monin–Obukhov similarity (Garratt 1992).

For the experiments described hereafter, the radiative transfer has been replaced by a Newtonian cooling, relaxing the atmosphere to a uniform temperature of 200 K over a timescale of 40 days. This provides a simple way to produce radiative cooling rates of the right magnitude while letting the tropopause level be determined by the dynamics alone. A two-dimensional, horizontally periodic domain is represented by a 320 × 78 grid. Horizontal resolution is 2 km. Vertical resolution varies from 50 m near the surface to 500 m at higher levels. The upper boundary is at height $z = 32$ km. A mean zonal wind profile is imposed with $\overline{u}(z) = \omega \min(z, z_c)$ with $\omega = 10^{-3}$ s^{-1} and $z_c = 5$ km. This is necessary

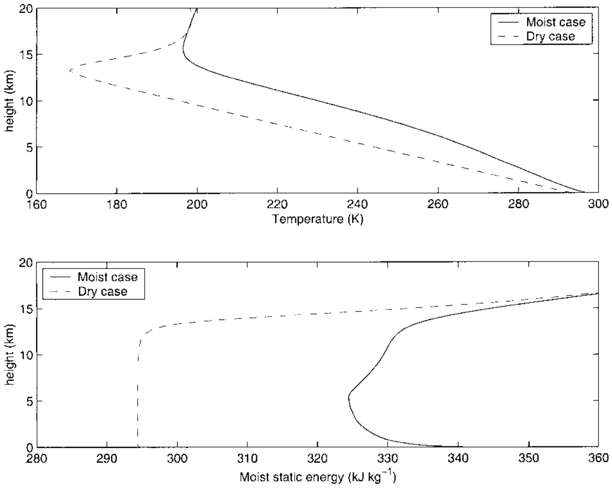

FIG. 2. (top) Horizontally averaged temperature in the moist (continuous line) and dry experiments (dashed line). (bottom) Horizontally averaged moist static energy in the moist (continuous line) and dry experiments (dashed line).

in two-dimensional simulations to avoid a quasi-biennial oscillation–like behavior due to the absorption of gravity waves in the stratosphere as discussed in Held et al. (1993). A sponge-layer is present in the top 10 km of the model to dissipate the gravity waves before they are reflected at the upper boundary. This model differs from that used by PBH in that it is two-dimensional, in that the explicit radiative transfer has been replaced by Newtonian cooling, and in that it uses Monin–Obukhov drag coefficients.

We analyze here two experiments: 1) dry convection, in the absence of water, and 2) moist convection, which includes the full microphysical parameterization of the model. The atmosphere reaches statistical equilibrium in less than 20 days in the dry case, and 40–60 days in the moist case. In the dry experiment, the Newtonian cooling is unchanged. This is not meant as a realistic model of dry radiative–convective equilibrium, but as a way of generating a model with a simpler entropy budget.

The vertical profiles of temperature and moist static energy $h = C_pT + L_vq + gz$ are shown in Fig. 2. In both the dry and moist simulations, the atmosphere is separated into the troposphere, where convection is active and the temperature distribution corresponds to an adiabatic profile, and the stratosphere, which is (approximately) in radiative equilibrium. In the dry case, there is a strong inversion at the tropopause that is reminiscent of the trade wind inversion in the subtropics. The stratospheric temperature minimum in the moist simulation is an artifact due to the downward heat flux associated with the damping of gravity waves by the sponge layer.

Convective activity differs greatly between the dry and moist cases. Figures 3 and 4 show snapshots of the convective activity for the dry and moist cases respec-

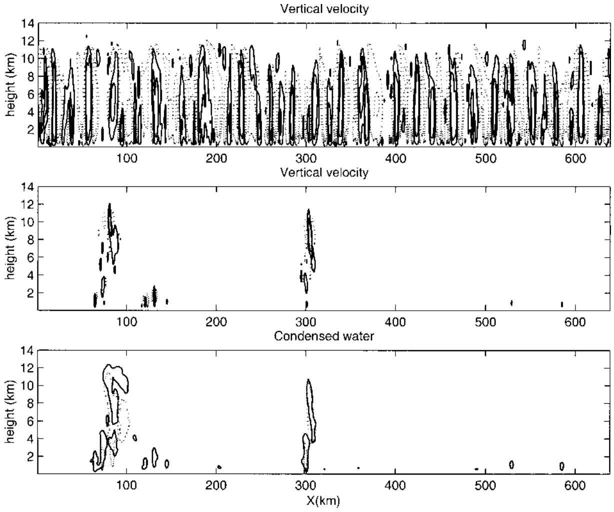

FIG. 3. Snapshots of the vertical velocity field in the (top) dry and (middle) moist experiments. Contours indicate vertical velocity of -3, -1, 1, and 3 m s^{-1}. Dotted lines are for negative values. (bottom) Snapshot of the cloud water and precipitating water: dotted lines indicate a precipitating water (snow and rain) content larger than 0.1 g kg^{-1}; continuous lines indicate cloud water content greater than 0.5 g kg^{-1}.

tively. In the dry case, convective cells cover the whole domain. The static energy is well homogenized with differences less than 1 kJ kg^{-1} between the updrafts and downdrafts. Maximum vertical velocity is about 8 m s^{-1} in the updraft and about 6 m s^{-1} for the downdrafts. In contrast, moist convection is more sporadic and tends to organize in a way reminiscent of squall lines. There are on average between 1 and 3 deep convective cells reaching up to the tropopause level. The strongest convective events have updrafts velocity up to 10–15 m s^{-1}, and are also associated with relatively strong downdrafts. In addition to deep convective clouds, there are also a large number of convective clouds detraining below 5 km. The horizontal variations of moist static energy are large, about 15 kJ kg^{-1}, in comparison with the dry case, indicating that individual updrafts carry a larger amount of energy in moist convection than in dry convection.

In the dry case, convection redistributes the sensible heat flux from the surface through the whole troposphere. Hence, radiative cooling is balanced by the resolved convective transport of sensible heat except in the surface layer. For moist convection, the radiative cooling is mostly balanced by diabatic heating, as shown in Fig. 5. The convective transport of sensible heat accounts for a small residual. The net latent heating can be separated into the latent heat release due to condensation and the latent cooling due to reevaporation, as shown in Fig. 6. The precipitation efficiency ϵ_p defined

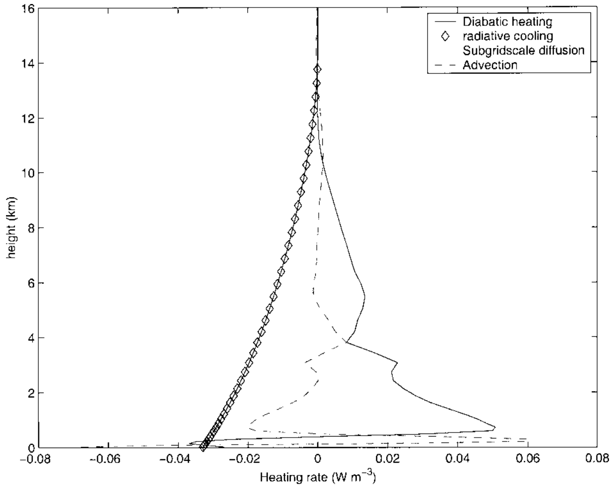

FIG. 5. Horizontally averaged heating rates.

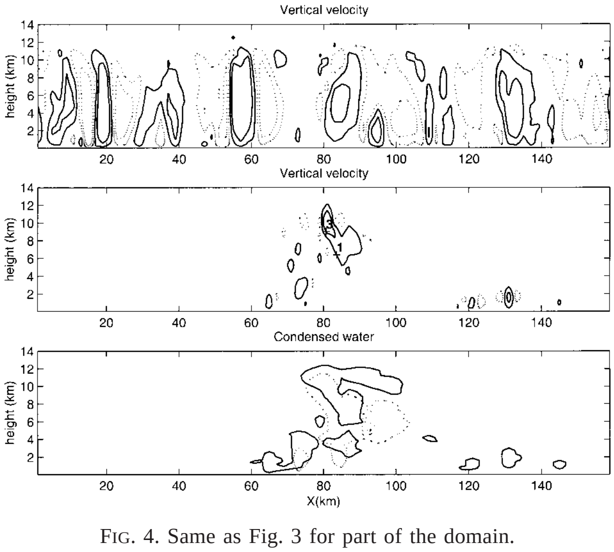

FIG. 4. Same as Fig. 3 for part of the domain.

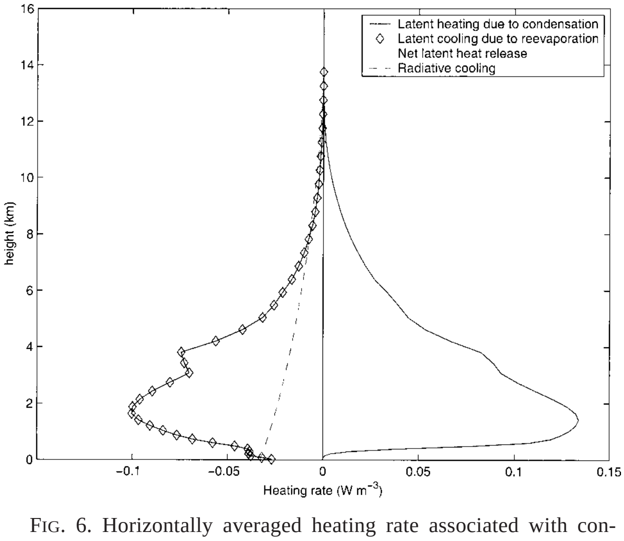

FIG. 6. Horizontally averaged heating rate associated with condensation, reevaporation, all phase changes (condensation and reevaporation), and radiative cooling.

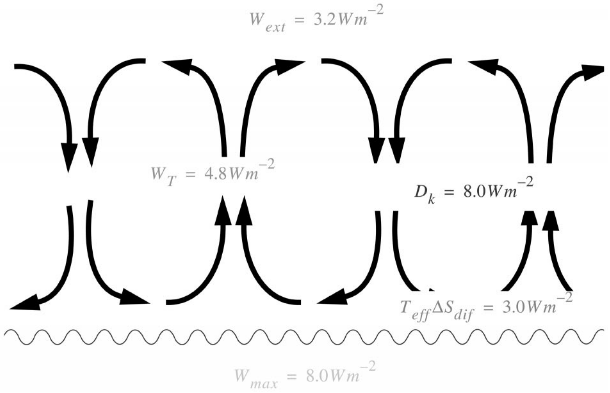

FIG. 7. Schematic representation of the mechanical energy and entropy budgets in the dry experiment. Here, $W_{\max}$ is the maximum theoretical work defined in (A8), W_T is the buoyancy flux, $W_{\rm ext}$ is the mechanical energy extracted from the mean shear, D_k is the turbulent dissipation, and $\Delta S_{\rm dif}$ is the irreversible entropy production due to diffusion of heat. The entropy budget for the numerical model is given in terms of Eq. (A8). The effective temperature is given by $T_{\rm eff} = T_{\rm surf} = 298.15$ K.

as the ratio of the precipitation rate at the surface to the total condensation is fairly small:

$$\epsilon_p = \frac{\int_\Omega C - E}{\int_\Omega C} \approx 0.27. \tag{26}$$

a. Mechanical energy budget

We turn now to the energy and entropy budgets of these simulations. The results of the analysis of the mechanical energy and entropy budget are schematically presented in Figs. 7 and 8. All quantities have been divided by the horizontal area of the domain so that they are expressed in watts per squared meter. The energy, mechanical energy and entropy budgets are also shown in Table 1–3. The various approximations used in the model and their consequences for the energy and entropy budgets are discussed in greater detail in the appendix.

Within the model's framework, the production of kinetic energy at the convective scales is given by the buoyancy flux,

$$W_b = \int_\Omega \overline{\rho} g w\left[\frac{T'}{\overline{\overline{T}}} + \left(\frac{R_v}{R_d} - 1\right)q_v - q_l\right], \tag{27}$$

where q_l is the mass of condensed water per unit mass of moist air. Here, an overline indicates a horizontally averaged quantity while a prime indicates the departure from the horizontal mean. As discussed by PBH, the buoyancy flux does not account for the total mechanical work done by convection. It does not include the work that is required to lift water and that is dissipated during precipitation. In statistical equilibrium, the frictional dissipation due to precipitation is given by (6). The total mechanical work in the system is

$$W = D_p + W_b = \int_\Omega \overline{\rho} g w\left(\frac{T'}{\overline{\overline{T}}} + \frac{R_v}{R_d}q_v\right). \tag{28}$$

The mechanical work can be written as $W = W_T + W_v$ where W_T is the thermal part of the buoyancy flux:

$$W_T = \int_\Omega \overline{\rho} g w \frac{T'}{\overline{\overline{T}}}, \tag{29}$$

and W_v is the contribution related to the vertical transport of water vapor:

$$W_v = \int_\Omega \overline{\rho} g w \frac{R_v}{R_d} q_v. \tag{30}$$

This decomposition is motivated by the relationship between water vapor transport and entropy production by moist processes, as discussed in PH.

The mean zonal wind $\overline{u}$ is fixed in the simulations. Maintaining this prescribed zonal wind against mixing by convection is equivalent to adding or subtracting kinetic energy. This external work is equal to

$$W_{\rm ext} = \int_\Omega \overline{\rho}\,\overline{U}\partial_z \overline{u'w'}. \tag{31}$$

The total generation of mechanical energy is thus the sum of three terms: thermal buoyancy flux W_T, water vapor buoyancy flux W_v and the external large-scale kinetic energy forcing $W_{\rm ext}$. The dissipation of mechanical energy associated with the turbulent cascade to smaller scales D_k is computed in the model by

$$D_k = \int_\Omega \overline{\sigma_{ij}}\partial_j V_i, \tag{32}$$

where $\overline{\sigma_{ij}}$ is the stress tensor due to subgrid-scale eddies. Hence, the mechanical energy budget in the numerical experiments is

$$W_T + W_v + W_{\rm ext} = D_p + D_k. \tag{33}$$

The mechanical energy budget of the dry experiment is schematically represented in Fig. 7. Convection generates $W_T = 4.8$ W m^{-2}, while eddies extract $W_{\rm ext} = 3.2$ W m^{-2} from the mean zonal wind. This large value of $W_{\rm ext}$ is an indication of the strong mixing associated with intense dry convection. The fact that $W_{\rm ext}$ is positive is not self-evident, given that the most unstable normal modes for two-dimensional Benard convection in shear flows produce countergradient momentum fluxes. Given that the eddies are able to extract a significant amount of energy from the mean flow, the dry simulation is better viewed as a mixture between convection and mechanically forced turbulence. Mechanical work is balanced by subgridscale dissipation $D_k = 8.0$ W m^{-2}.

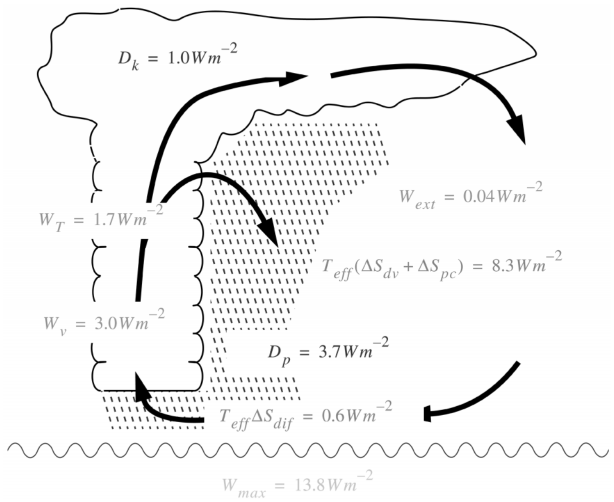

FIG. 8. Schematic representation of the mechanical energy and entropy budgets in the moist experiment. Here, $W_{\max}$ is the maximum theoretical work defined by (A12), W_T is the thermal contribution to the buoyancy flux, W_v is the buoyancy flux associated with water vapor transport, W_{ext} is the mechanical energy extracted from the mean shear, D_k is the turbulent dissipation, D_p is the precipitation-induced dissipation, ΔS_{dif} is the irreversible entropy production due to diffusion of heat, and $\Delta S_{\text{dv}} + \Delta S_{\text{pc}}$ is the irreversible entropy production due to the moistening effect. The entropy budget for the numerical model is given in terms of Eq. (A12). The effective temperature is given by $T_{\text{eff}} = T_v = 281.3$ K.

The results for the moist experiment are shown in Fig. 8. Most of the mechanical energy is produced by water vapor flux as $W_v = 3.0$ W m^{-2}. The thermal buoyancy flux accounts for only $W_T = 1.7$ W m^{-2}. Because of the intermittent character of moist convection, mixing of momentum is much weaker than in the dry case. Only $W_{\text{ext}} = 0.04$ W m^{-2} is extracted from the mean wind. Dissipation is dominated by precipitation, which accounts for $D_p = 3.7$ W m^{-2}. The subgrid-scale dissipation of kinetic energy is only $D_k = 1.0$ W m^{-2}.

b. Energy budget

In radiative–convective equilibrium, radiative cooling in the troposphere is balanced by the surface heat fluxes. However, in the numerical model, energy is not exactly conserved. This results primarily from the fact that frictional heating is not included in the potential temperature equation used in the model. In the appendix, it is shown that this nonconservation translates into spurious heat sources. In both the dry and moist case, nonconservation of energy translates in a spurious heat sink equal to $-W_T$. The additional error term in Table 2 is due to the change in internal energy of the system.

In the dry case, the sensible heat flux at the surface is 111.7 W m^{-2}. It is balanced by the Newtonian cooling of 106.9 W m^{-2} and by the spurious cooling of 4.8 W m^{-2}.

The troposphere is warmer in the moist case than in the dry case. This explains that the Newtonian cooling is significantly larger, with a net cooling of 157.4 W m^{-2}. The latent heat flux is 143.3 W m^{-2}, which is much larger than the sensible heat flux of 16.5 W m^{-2}. There is also an additional spurious heat sink of 1.7 W m^{-2}.

TABLE 1. Mechanical energy budget in the numerical simulations [cf. Eq. (33)] with the buoyancy flux due to the sensible heat transport W_T, buoyancy flux due to water vapor transport W_v, mechanical energy extracted from the mean shear W_{ext}, frictional dissipation resulting from turbulent cascade D_k, and precipitation-induced frictional dissipation D_p. Values are given in W m^{-2}.

	W_T	$+ W_v$	$+ W_{\text{ext}}$	$= D_k$	$+ D_p$
Dry case	4.8	+0.0	+3.2	=8.0	+0.0
Moist case	1.7	+3.0	+0.04	=1.0	+3.7

c. Entropy budget

The nonconservation of energy also modifies the entropy budget, by introducing additional entropy sources and sinks associated with the spurious heat sources and sinks. It is shown in the appendix that these changes result in only a small error in the total irreversible entropy production in the system.

The entropy budget is analyzed by comparing the maximum theoretical work to the work effectively performed by convection and to the other irreversible entropy sources:

$$W_{\max} = W_T + W_v + T_{\text{eff}}\Delta S_{\text{dif}} + T_{\text{eff}}(\Delta S_{\text{dv}} + \Delta S_{\text{pc}}). \quad (34)$$

Nonconservation of energy has two consequences for Eq. (34). First, the definition of the maximum work must be slightly modified. We use the expression (A8) for the dry case and expression (A12) for the moist case. Second, the contribution of the nonfrictional irreversible entropy source is multiplied by an effective temperature T_{eff}, instead of the frictional temperature T_d. In the dry case, T_{eff} is equal to the surface temperature T_{surf}, and, in the moist case, it is equal to the effective temperature of water vapor transport T_v. The latter is defined by

$$T_v = \left(\int_\Omega \frac{\overline{\rho} w q_v}{T}\right)^{-1} \int_\Omega \overline{\rho} w q_v. \quad (35)$$

By making these changes, the errors due to spurious numerical heat sources are incorporated in $W_{\max}$ and T_{eff}, so that the various irreversible entropy sources can still

TABLE 2. Energy budget in the numerical simulations [cf. Eqs. (A5) and (A9)], with the surface sensible heat flux Q_{sen}, surface latent heat flux Q_{lat}, radiative cooling Q_{rad}, and spurious heat source in the model Q_{sp}. The imbalance *err* in the energy budget is mostly due to changes in the internal energy of the atmosphere. Values are given in W m^{-2}.

	Q_{sen}	$+ Q_{\text{lat}}$	$+ Q_{\text{rad}}$	$+ Q_{\text{sp}}$	$= err$
Dry case	111.7	+0	−106.9	−4.8	= +0.1
Moist case	16.5	+143.3	−157.4	−1.7	= +0.7

TABLE 3. Entropy budgets in the numerical simulations, with maximum work W_{max} buoyancy flux due to sensible heat flux W_T, buoyancy flux due to water vapor transport W_y, moistening effect $T_v \Delta S_{dv} + \Delta S_{pc}$, diffusion of heat by subgrid-scale eddies $T_v \Delta S_{dif}$, and imbalance in the budget *err*. Values are given in W m^{-2}. The effective temperature T_{eff} is equal to the surface temperature $T_{eff} = T_{surf} = 298.15$ K in the dry case, and to the effective temperature of the water vapor transport $T_{eff} = T_v = 281.3$ K in the moist case.

	W_{max}	$= W_T + W_v$	$+ T_{eff} (\Delta S_{dv} + \Delta S_{pc})$	$+ T_{eff} \Delta S_{dif}$	$+ err$
Dry case	8.0	=4.8	+0	+3.0	+0.2
Moist case	13.8	=4.7	+8.3	+0.6	+0.2

be compared through the budget (34). As discussed in the appendix, the spurious heat sources do not significantly affect entropy budget as long as the spurious heat sources remain small in comparison to the radiative cooling.

The irreversible entropy production due to diffusion of heat ΔS_{diff} is obtained from (16), after replacing the molecular sensible heat flux by the subgrid-scale flux. The sensible heat transport can be viewed as resulting from subgrid-scale eddies. These eddies are able to extract available potential energy and convert it into kinetic energy, which is in turn dissipated through friction. The entropy production ΔS_{diff} in the model is therefore partially due to molecular diffusion of heat and partially due to the generation and dissipation of kinetic energy by subgrid-scale eddies. It is, however, not possible to separate ΔS_{diff} between these two processes without making further assumptions.

The entropy production due to moist processes $\Delta S_{dv} + \Delta S_{pc}$ is obtained through formula (21). The latent heat transport can be obtained directly from the model. The work performed by water vapor expansion cannot be obtained directly, but the ratio W_{vap}/T_{vap} in (21) can be approximated by W_v/T_v, as shown in the appendix of PH. This allows us to determine the corresponding entropy production without requiring the explicit knowledge of the diffusive flux of water vapor or the relative humidity.

As the specific humidity of an air parcel is conserved in the absence of phase changes and diffusion of water vapor, the entropy production due to the mixing of two air masses is equal to the entropy production that would result from the diffusion of the water vapor from one air mass to the other. Turbulent motion can only increase the gradient of specific humidity, but cannot change the specific humidity distribution. The final homogenization between the two air masses must be *in fine* accomplished through molecular diffusion of water vapor. Subgrid-scale eddies cannot change the entropy production due to moist processes if they are not associated with phase changes or result in a vertical transport of water vapor. This implies that, although the numerical model uses a subgrid-scale parameterization for the diffusion of water vapor, the entropy production for $\Delta S_{dv} + \Delta S_{pc}$ computed in the simulations corresponds indeed to molecular diffusion and irreversible phase changes.

In the dry case, the maximum work is $W_{max} = 8.0$ W m^{-2}, while the work done by convective systems is only $W_T = 4.8$ W m^{-2}. A direct estimate of the contribution of the subgrid-scale diffusion of sensible heat yields $T_{eff}\Delta S_{diff} = 3.0$ W m^{-2}. As discussed earlier, this entropy production is due both to molecular diffusion of heat and to the generation and dissipation of kinetic energy by subgrid-scale eddies. There is also a small imbalance in the entropy budget that can be attributed to small changes in the total entropy of the system and to numerical errors in the averaging method and advection scheme.

In the moist experiment, the maximum work is $W_{max} = 13.8$ W m^{-2}. This value is larger than in the dry case mainly because the total heat flux is larger. The work effectively performed by convection $W_T + W_v = 4.7$ W m^{-2} is approximately one-third of this theoretical maximum. Subgrid-scale diffusion of heat accounts only for a small portion of the irreversible entropy production with $T_{eff}\Delta S_{diff} = 0.6$ W m^{-2}. The irreversible entropy production due to moist processes is obtained through (21) and accounts for most of the entropy production, with $T_{eff}(\Delta S_{dv} + \Delta S_{pc}) = 8.3$ W m^{-2}. There is an imbalance $\sim$0.2 W m^{-2} in the entropy budget (34), which can be attributed to numerical artifacts.

These simulations indicate that the entropy budgets of dry and moist convection differ significantly. On the one hand, dry convection can be described as acting mostly as a heat engine: most of the entropy production is due to frictional dissipation. (As discussed earlier, part of the entropy production by subgrid-scale diffusion of heat should be interpreted as frictional dissipation by unresolved eddies.) On the other hand, the entropy budget of moist convection is more complex. The largest

TABLE 4. The nondimensional version of the entropy budget is obtained by dividing the dimensional budget (34) by the maximum theoretical work W_{max}. The nondimensional parameters λ, β, μ, η_k, and η_{max} are defined in the text. The value of these nondimensional quantities in the moist simulations are also shown. In the moist case, we have $\lambda = 0.82$, $\beta = 0.27$, $\mu = -0.06$, $\eta_k = 0.006$, and $\eta_{max} = 0.087$.

Entropy budget	W_{max}	$= W_T + W_v$	$+T_d(\Delta S_{dv} + \Delta S_{pc})$	$+T_d\Delta S_{dif}$
Nondimensional analysis	1	$= 1 - \lambda(1 - \beta)$	$+\lambda(1 - \beta)$	+0
Simulations	1	= 0.34	+0.60	+0.04

fraction of the entropy production is due to irreversible phase changes and diffusion of water vapor. Together, these account for about 60% of the total entropy production. Frictional dissipation still accounts for a significant fraction of the irreversible entropy production. However, most of the frictional dissipation occurs in the shear zones surrounding falling hydrometeors, which represents about 30% of the total entropy source. The amount of kinetic energy generated at the convective scales and dissipated through turbulence only accounts for a small fraction of the total entropy production.

An important question is to what extend the entropy budget of our simulations is representative of that of the physical atmosphere. One may argue that the model's resolution is insufficient to accurately simulate the behavior of convective updrafts, or that the subgrid-scale diffusion and dissipation do not properly represent turbulent mixing and dissipation, or that the bulk parameterization is inadequate. Similarly, one may ask whether convective organization has an impact on the entropy budget, and thus whether the use of a two-dimensional domain has an impact on our results. Notice that Shutts and Gray (1999) also found that the amount of work that was generated in three-dimensional simulations of moist convection was significantly smaller than that predicted by the perfect heat engine of RI and EB. This is also the case in the three-dimensional simulations discussed in PBH. However, to fully address these questions, one would require more extensive comparisons between numerical models and observations. Unfortunately, if it is relatively easy to analyze the entropy budget of numerical simulations, it is extremely difficult to do this for the physical atmosphere. In fact, we do not know of any observational study that can be used to fully determine the entropy production by the various irreversible processes.

There is however an aspect of moist convection that leads us to believe that the entropy budget of our moist simulation is representative of a more general behavior: the fact that the irreversible entropy production by moist processes and the expansion work by water vapor are directly related to the latent heat transport, as discussed in PH. The important contribution of these two terms in the entropy and mechanical energy budgets can deduced directly from the observation that the convective heat transport is mostly due to latent heat, as seen in Fig. 5 for our simulations. In such cases, moist convection acts more as an atmospheric dehumidifier than as a heat engine, and irreversible phase changes and diffusion of water accounts for a large fraction of the total irreversible entropy production. The moist simulation can thus be view as illustrative of the entropy and mechanical energy budget of an atmosphere in radiative–convective equilibrium in which the convective heat transport is dominated by the latent heat transport.

4. Nondimensional analysis of the entropy budget

The entropy budget of moist convection is now discussed in terms of nondimensional parameters. Of particular interest is the estimation of a convective efficiency η_k defined as the ratio of the turbulent dissipation D_k to the radiative cooling $Q_{\rm rad}$:

$$\eta_k = \frac{D_k}{|Q_{\rm rad}|}. \tag{36}$$

The convective efficiency measures the amount of mechanical work effectively used to generate convective motions. It is the parameter required in the theory of RI, EB, or Craig (1996) to determine the vertical velocity of convective systems. In contrast to RI and EB, we argue here that η_k is not equal to the thermodynamic efficiency of a perfect heat engine, but depends crucially on the latent heat transport by convection and on the formation of precipitation.

The entropy sink associated with differential heating can by measured in terms of the maximum work $W_{\rm max}$. The maximum theoretical mechanical efficiency $\eta_{\rm max}$ is the ratio between $W_{\rm max}$ and the net radiative cooling $Q_{\rm rad}$. From (14), this nondimensional coefficient is

$$\eta_{\rm max} = \frac{W_{\rm max}}{|Q_{\rm rad}|} = T_d\left(\frac{1}{T_{\rm rad}} - \frac{1}{T_{\rm surf}}\right) \approx \frac{T_{\rm surf} - T_{\rm rad}}{\overline{\overline{T}}}. \tag{37}$$

The maximum theoretical efficiency $\eta_{\rm max}$ is approximately equal to the thermodynamic efficiency of a Carnot cycle.

The thermodynamic action of the latent heat transport can be measured by a quantity $W_{\rm lat}$ introduced by PH and defined as the amount of work that would be performed if the latent heat is transported by a perfect heat engine:

$$W_{\rm lat} = Q_{\rm lat}T_{\rm vap}\left(\frac{1}{T_{\rm lat}} - \frac{1}{T_{\rm surf}}\right). \tag{38}$$

Equation (21) indicates that the latent heat transport is related to the entropy production due to moist processes and the expansion work of water vapor. We introduce here a nondimensional parameter λ as the ratio between $W_{\rm lat}$ and $W_{\rm max}$:

$$\lambda = \frac{W_{\rm lat}}{W_{\rm max}} \approx \frac{T_{\rm surf} - T_{\rm lat}}{T_{\rm surf} - T_{\rm rad}}\frac{Q_{\rm lat}}{|Q_{\rm rad}|}. \tag{39}$$

This nondimensional parameter measures the contribution of latent heat transport relative to the total heat transport. It is shown in PH that the ratio of the expansion work by water vapor $W_{\rm vap}$ to $W_{\rm lat}$ is approximately given by

$$\beta = \frac{W_{\rm vap}}{W_{\rm lat}} \approx \frac{H_{e_s}}{H_p}, \tag{40}$$

where $H_{\rm p} = \partial_z(\ln p)^{-1}$ is the pressure scale height, and $H_{e_s} = \partial_z(\ln e_s)^{-1}$ is the scale height for the saturation water vapor pressure. In the Tropics, we have $H_p \sim 8$ km and $H_{e_s} \sim 2$ km; hence, the value of β should be small: $0.2 < \beta < 0.3$. Dividing (21) by $W_{\rm max}$ and using (39) and (40) yields

$$\frac{\overline{T}(\Delta S_{dv} + \Delta S_{pc})}{W_{max}} \approx \lambda(1 - \beta). \quad (41)$$

If, as discussed in section 2, the entropy production results primarily from frictional dissipation and moist processes (25), then the work performed by moist convection can be derived by using (37) and (41) into (15):

$$\frac{W}{|Q_{rad}|} = \frac{D_p + D_k}{|Q_{rad}|} \approx \eta_{max}[1 - \lambda(1 - \beta)]. \quad (42)$$

We introduce another parameter γ equal to the ratio of the total dissipation associated with precipitation to the mechanical work done by the water vapor expansion:

$$\gamma = \frac{D_p}{W_{vap}} \approx \frac{D_p}{W_v} = \frac{\int_\Omega \rho(q_v + q_l)w}{\int_\Omega \rho \frac{R_v}{R_d} q_v w}. \quad (43)$$

The approximation $W_{vap} \approx W_v$ is derived in the appendix of PH. The transport of condensed water by air motions is likely to be dominated by the updrafts. Hence it is expected that $\int_{z=z_0} \rho q_l w > 0$, so that the vertical transport of water vapor gives a lower bound on the frictional dissipation due to precipitation. The constraint implies that $\gamma \geq 0.6$. In the numerical simulation presented in this paper, we find $\gamma \approx 1.2$. The difference between the work due to water vapor expansion and the dissipation associated with precipitation is the amount of kinetic energy that is produced by the hydrological cycle and that is available to produce atmospheric motion. This can be measured by the parameter μ defined as

$$\mu = \frac{W_{vap} - D_p}{W_{lat}} \approx \beta(1 - \gamma). \quad (44)$$

When the dissipation due to precipitation is larger than the expansion work of water vapor, $\gamma \geq 1$, the hydrological cycle dissipates more mechanical energy than it produces and μ is negative. Both terms in the product on the right-hand side are small, and we expect μ to be small also, albeit its sign is uncertain. In the simulation we obtain $\mu \approx -0.06$.

The convective efficiency η_k is defined as the ratio of the frictional dissipation due to turbulent cascade to the total heat flux. It is given by

$$\eta_k = \frac{D_k}{|Q_{rad}|} \approx \eta_{max}[\lambda\mu + (1 - \lambda)]. \quad (45)$$

The convective efficiency measures the amount of mechanical work effectively used to generate convective motion. It is the parameter that is required in the theories of RI, EB, or Craig (1996) to determine the vertical velocity of convective systems. For a given distribution of heat sources and sinks, the convective efficiency depends on μ and λ. Tables 4 and 5 show nondimensional versions of the mechanical energy and entropy budgets, which have been obtained by dividing the dimension counterpart by W_{max}.

TABLE 5. The nondimensional expression for the mechanical energy budget is obtained by dividing the dimensional budget by W_{max}. The nondimensional parameters λ, β, μ, η_k are defined in the text.

Mechanical energy budget	W_T	$+W_v$	$= D_k$	$+D_p$
Nondimensional analysis	$1 - \lambda$	$+\lambda\beta$	$= \frac{\eta_k}{\eta_{max}}$	$+\lambda(\beta + \mu)$
Simulations	0.12	+0.22	= 0.07	+0.27

Equation (45) indicates that the convective efficiency of moist convection is sensitive to the relative importance of the latent heat transport and the precipitation-induced dissipation. The exact values of λ and μ may be sensitive to a wide variety of factors, such as wind shear, microphysics, or radiative cooling. It is also conceivable that these parameters could be determined from simple external constraints. We examine now a few alternative for such closure and investigate how this would affect the behavior of moist convection.

The expression (45) could be simplified by assuming that the expansion work due to water vapor is balanced by the frictional dissipation due to precipitation:

$$\mu \approx 0. \quad (46)$$

In this case, the convective efficiency is then given by

$$\eta_k \approx (1 - \lambda)\eta_{max}. \quad (47)$$

The convective efficiency is primarily determined in this case by the relative importance of latent heat transport.

If the latent heat transport accounts for only a small portion of the total convective heat transport $\lambda = 0$, then the convective efficiency is equal to the maximum efficiency $\eta_k = \eta_{max}$, as in RI and EB. This requires the effective temperature of latent heat release to be close enough to the surface temperature $T_{surf} - T_{lat} \ll T_{surf} - T_{rad}$, or equivalently, that most of the convective heat transport is due to sensible heat. As discussed in PH, this is theoretically possible in the case of low precipitation efficiency and strong saturated downdrafts. It is, however, unlikely for the current tropical condition. For the sensible heat transport to be of the same order of magnitude as the latent heat, the ratio T'/q' must be of the order of 2.5 K/(g kg^{-1}), which is much larger than the observed ratio in tropical convection.

When the heat transport by convection is mostly due to latent heat—when λ is close to 1—moist convection will have the following characteristics: 1) irreversible phase changes and diffusion of water vapor are the main irreversible entropy sources; 2) the expansion work of water vapor accounts for a large fraction of the total mechanical work in the system; 3) precipitation-induced dissipation accounts for a significant portion of the total dissipation; and 4) the convective efficiency is much smaller than the efficiency of a perfect heat engine.

Unfortunately, a large value of λ makes it more difficult to obtain a theory for convective efficiency. Indeed, for large λ, the right-hand side of (45) becomes strongly dependent on both λ and μ. As turbulent dissipation accounts for a very small fraction of the entropy production, it is very sensitive to small changes in the other irreversible sources of entropy.

This also suggests some caution as to whether or not the convective efficiency could actually be determined from the entropy budget. For instance, one can consider an alternative closure by assuming that precipitation-induced dissipation accounts for most of the frictional dissipation: $D_p \gg D_k$. This is equivalent to taking $\eta_k \approx 0$ in (45) and provides a relationship between λ and μ:

$$\mu = -\frac{1 - \lambda}{\lambda}. \tag{48}$$

In this case, γ is related to the latent heat transport by

$$\gamma = 1 + \frac{1 - \lambda}{\beta\lambda}. \tag{49}$$

For such a closure, the entropy budget provides a constraint on the frictional dissipation due to precipitation but cannot be used to determine the turbulent dissipation D_k. The buoyancy of the updrafts is limited by the condensate loading, and the vertical velocity of the convective system should be derived from the microphysical processes associated with the formation of precipitation in convective updrafts.

5. Conclusions

The entropy budget of an atmosphere in radiative–convective equilibrium is characterized by a balance between an entropy sink due to the differential heating of the atmosphere and the entropy production due to the various irreversible processes associated with convection. The main issue is to determine the relative contribution of each irreversible process. For moist convection, one must essentially distinguish between frictional dissipation on the one hand, and irreversible phase changes and diffusion of water vapor on the other. This can be recast as to whether convection behaves more as a heat engine or as an atmospheric dehumidifier. This question is important not only for a theoretical understanding of how the second law of thermodynamics applies to a convective atmosphere, but also because it may give a better insight into the behavior of convection. This is the approach followed by RI and EB, who derive vertical velocity, intermittency, and CAPE of moist convection based on the assumption that the turbulent dissipation of convective updrafts is the main irreversible mechanism associated with moist convection.

Our results differ significantly from EB and RI in that we argue that phase changes, diffusion of water vapor, and frictional dissipation induced by precipitation, reduce the amount of kinetic energy generated at the convective scales. In numerical simulations of radiative–convective equilibrium with a CEM, moist convection has four major characteristics: 1) irreversible phase changes and diffusion of water vapor are the main irreversible source of entropy; 2) most of the frictional dissipation occurs in the microscopic shear zones surrounding falling precipitation; 3) water vapor expansion is the main source of mechanical work; and 4) turbulent dissipation cascading from convective scale eddies accounts only for a small fraction of the total entropy production.

These characteristics are directly related to the fact the convective heat transport is mostly due to the latent heat transport. Indeed, PH show that, in an atmosphere in radiative–convective equilibrium, the entropy production by phase changes and diffusion of water vapor is related to the latent heat transport by convection and the expansion work by water vapor. This indicates that the entropy budget in the numerical simulations is representative of the entropy budget of an atmosphere where a significant portion of the convective heat transport is due to latent heat. In such cases, tropical convection acts more as an atmospheric dehumidifier than as a heat engine.

The entropy budget can be used to determine a convective efficiency, defined as the ratio of the generation of kinetic energy at the convective scales to the total heat transport by convection, in a similar way to the approach followed by Craig (1996). Convective efficiency depends primarily on how the heat transport is separated between latent and sensible heat, and, to a lesser extent, on the magnitude of the precipitation-induced dissipation. The theories of RI and EB correspond to the case where latent heat transport by convection is very small. Although this is in theory possible, it seems unlikely for tropical convection. Some uncertainties remain, mostly associated with microphysical processes. It is not clear at this point if a theory as simple as the one proposed by RI and EB could be obtained for moist convection. A first step toward such a theory should be to assess the factors that can influence the convective transport of latent heat and the precipitation-induced dissipation. Further research in this direction could potentially lead to a quantitative theory for CAPE and vertical velocity in moist convection.

Acknowledgments. We would like to thank V. Balaji, Leo Donner, Kerry Emanuel, and Steve Klein for their helpful conversations on this topic.

APPENDIX

Spurious Heat Sources and Entropy Budget in Numerical Models

Approximations made in numerical models result in the fact that the model's entropy and energy budgets are different from the energy and entropy budgets of a physical atmosphere. For example, frictional heating is often neglected in CEMs. These differences are usually small in comparison to radiative cooling, but can nevertheless be problematic since the model violates the first law of thermodynamics. One must therefore address the question of whether or not these errors have an impact on the problem at hand. We discuss here how errors related to the conservation of energy modify the irreversible entropy production of a system.

Consider a numerical model for which some terms in the internal energy equation have been neglected. This can be treated as if there were a spurious heat source Q_{sp} that compensates exactly for the missing terms. For example, neglecting the frictional heating is equivalent to $Q_{sp} = -D$. The energy budget in the model is

$$Q_{sen} + Q_{lat} + Q_{rad} + Q_{sp} = 0. \quad \text{(A1)}$$

For a model based on prognostic equations for momentum and internal energy, entropy is handled implicitly. If an approximation results in a spurious heat source, it also results in a spurious entropy source equal to the spurious heat source divided by an effective temperature T_{sp}. Thus, the entropy budget is

$$\frac{Q_{sen} + Q_{lat}}{T_{surf}} + \frac{Q_{rad}}{T_{rad}} + \Delta S_{irr} + \frac{Q_{sp}}{T_{sp}} = 0. \quad \text{(A2)}$$

Dividing (A1) by T_{surf} and removing it from the entropy budget (A2) yields the expression for the irreversible entropy production in the model:

$$\Delta S_{irr,mod} = |Q_{rad}|\left(\frac{1}{T_{rad}} - \frac{1}{T_{surf}}\right) + Q_{sp}\left(\frac{1}{T_{surf}} - \frac{1}{T_{sp}}\right). \quad \text{(A3)}$$

The difference between the irreversible entropy production of the model and that of the physical atmosphere *for the same radiative cooling* is

$$\Delta S_{irr,mod} - \Delta S_{irr} = Q_{sp}\left(\frac{1}{T_{surf}} - \frac{1}{T_{sp}}\right). \quad \text{(A4)}$$

As long as the spurious heat sources are small in comparison to the total heat sink $|Q_{sp}| \ll |Q_{rad}|$, and as long as the effective temperature of the spurious heat source is close enough to the surface temperature $|T_{surf} - T_{sp}| \sim |T_{surf} - T_{rad}|$, the relative error in the irreversible entropy production is small. This is the case when one neglects the frictional heating or the expansion work by water vapor in the internal energy equation.

We discuss now the entropy and energy budgets of the model used for this paper.

a. Dry atmosphere

For the dry case, the only difference between the model atmosphere and the physical atmosphere resides in the model's neglect of the heating due to frictional dissipation D_k. This difference results in a spurious heat source $Q_{sp} = -D_k = -W_T$:

$$Q_{sen} + Q_{rad} - W_T = 0. \quad \text{(A5)}$$

The effective temperature of this spurious heat source is equal to the effective temperature of frictional dissipation. Hence, the entropy budget of the model is

$$\frac{Q_{sen}}{T_{surf}} + \frac{Q_{rad}}{T_{rad}} + \Delta S_{dif} = 0. \quad \text{(A6)}$$

The energy and entropy budgets (A5) and (A6) are similar to the energy and entropy budgets of a heat engine for which the work W_T is exerted on the environment, instead of being dissipated internally in the system. There are different ways of combining the two budgets (A5) and (A6) to derive a relationship between mechanical work and entropy sources and sinks. We choose to subtract (A5) from the entropy budget (A6) multiplied by T_{surf} to obtain

$$W_{max} = W_T + T_{surf}\Delta S_{dif}. \quad \text{(A7)}$$

In this budget, the maximum work W_{max} that could be performed by convection is defined by

$$W_{max} = \frac{T_{surf} - T_{rad}}{T_{rad}}|Q_{rad}|. \quad \text{(A8)}$$

This is equivalent to assuming that the frictional dissipation occurs at the surface temperature in (13) and (14).

b. Moist atmosphere

Lipps and Hemler (1982) argue that the current model's energy budget for a moist atmosphere can be approximated by

$$Q_{lat} + Q_{sen} + Q_{rad} = W_T. \quad \text{(A9)}$$

This results from two approximations. First, frictional heating $D_k + D_p$ is neglected. Second, although the effect of water vapor is included in the buoyancy, the effect of the virtual temperature is not properly included in the potential temperature equation. As a result, the work performed by water vapor is not balanced by a reduction of the internal energy of the system, which translates into a spurious heat source equal to W_v. These two approximations result in a spurious heat source $Q_{sp} = -D_k - D_p + W_v = -W_T$. In the entropy budget, the spurious cooling associated with omitting $D_k + D_p$ compensates for the irreversible entropy production due to frictional dissipation. The spurious heat source associated with W_v corresponds to an additional entropy source equal to W_v/T_v. The entropy budget of the numerical simulations can be written as

$$\frac{Q_{\rm lat} + Q_{\rm sen}}{T_{\rm surf}} + \frac{Q_{\rm rad}}{T_{\rm rad}} + \Delta S_{\rm dif} + \Delta S_{\rm dv} + \Delta S_{\rm pc} + \frac{W_v}{T_v} = 0. \tag{A10}$$

The energy and entropy budgets can be combined to obtain a relationship between the work done by convection and the different entropy sources and sinks. Subtracting the entropy budget (A10) multiplied by T_v from (A9) yields

$$W_{\rm max} = W_v + W_T + T_v \Delta S_{\rm dv} + \Delta S_{\rm pc} + T_v \Delta S_{\rm dif}, \tag{A11}$$

where the maximum theoretical work is given by

$$W_{\rm max} = \frac{T_{\rm surf} - T_v}{T_{\rm surf}}(Q_{\rm sen} + Q_{\rm lat}) + \frac{T_v - T_{\rm rad}}{T_{\rm rad}}|Q_{\rm rad}|. \tag{A12}$$

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