Introduction to Mathematical Modeling, Spring 2000

Assignment 5, due March 1.

1. The equation below represents an oscillator with highly nonlinear damping:

$$\ddot{x} = -\omega^2 x - \gamma \dot{x}^3 \quad . \tag{1}$$

a. With $\omega = 2$, $\gamma = 1$. $\dot{x}(0) = 2$, and $x(0) = 1$, estimate the first t with $\dot{x} = 1.5$ and the x value where this occurs. Do this by computing the Taylor series for $\dot{x}$ and x as a function of t keeping only terms up to order t (i.e., dropping all terms quadratic and higher in t).

b. Estiamte the error in your answer to part a by including terms up to quadratic in t, but not higher. You will have to differentiate the equation (1) with respect to t to get the required coefficients. When you have to solve a quadratic, do it approximately, but in a way that gets terms including order t^2 correct. This will involve using the Taylor series for the square root function.

c. Now keep $\omega = 2$, but take $\gamma = .05$, $\dot{x}(0) = .5$, and $x(0) = 0$. How long will it take until 90% of the energy has dissipated out of the oscillator? How long until 99% of the energy has dissipated away? Contrast these results to those for simple linear friction.

2. In a study of interactin between drugs, research administered varying doses of drugs A and B and measured responses $M1$, $M2$, and $M3$. Some of the data are in the table below.

Trial	A	B	$M1$	$M2$	$M3$
1	100	32	4.0	59	18.6
2	100	34	4.8	57	16.9
3	110	32	3.7	56	20.2

Make a linear approximation to the three responses to the two dose levels about $A = 100$, $B = 32$. Use this to find a dose that keeps $M2$ below 55, $M3$ below 20, and makes $M1$ as low as possible.

3. (corrected from problem 3 of homework 4) Construct a matrix, $M(t)$ so that

$$\begin{pmatrix} x(t) \\ \dot{x}(t) \end{pmatrix} = M(t) \begin{pmatrix} x(0) \\ \dot{x}(0) \end{pmatrix} \quad .$$

If everything goes right, the matrix M will have only real entries even though the intermediate quantities U, c, and λ are not real.

4. Here is yet another way an oscillator can lose energy. The mass, m is connected by a spring with spring constant k to a much larger mass, M. This large mass can slide over a surface, but with a large friction coefficient, Γ (Γ is the capital of γ).

a. Assume that M and Γ are large, and that the displacement of the smaller mass is given by $A(t)\sin(\omega t)$. Figure out how fast A decreases, approximately.

b. Express the dynamics of the small mass and large mass system in terms of a 4×4 matrix, A. Find the eigenvalues of A related to decaying oscillation of the smaller mass. See how well this exact result agrees with the approximate result from part a.