Assignment 2.

Given February 5, due January 12.

Objectives: To explore an ill conditioned matrix.

For this assignment you will need software that estimates the condition number of a matrix. This can be matlab, or a standard scientific subroutine package (e.g. NAG) or software from netlib. If you don't have access to such software, you will have to skip part (1). If you don't find a routine that estimates condition number but you can find one that computes the singular values or the singular value decomposition, the condition number is $\sigma_{\max}/\sigma_{\max}$. If you can't compute singular values but you can compute eigenvalues of symmetric matrices, compute the eigenvalues of A^*A, $\lambda_1 \geq \cdots \geq \lambda_n$. The condition number of A is $\sqrt{\lambda_1/\lambda_n}$.

For a fixed n, define A to be the $n \times n$ matrix with all entries zero except the $a_{ii} = 1$ and $a_{1,i+1} = 2$ for all i. That is, A has 1 on the diagonal and 2 just above the diagonal and all the rest zero. The determinant of this matrix is 1 and its norm is about 3 (depending on exactly which norm is used).

(1) Find the (estimated) condition number of A as a function of n for $n = 2, 3, \ldots$ until the condition number becomes (in the computer) infinite.

(2) Let $x \in R^n$ be the vector with n components whose components are given by $x_j = 1$ if j is even and $x_j = 1/3$ if j is odd. The value $1/3$ is not stored exactly in the computer in floating point. Compute $b = Ax$ by simple matrix multiplication. Now compute y by solving the equations $Ay = b$ where b has just been computed. Write a program to do this in FORTRAN, C, or C++ using back back substitution on an upper triangular matrix that is bidiagonal (see Kahaner, Moler, and Nash). Compute $\|x - y\|$ for some norm and plot this as a function of n so long as possible. Compute $|Ay - b|$ for the y you have and check its dependence on n. Comment on the contrast between this and the dependence of $\|x - y\|$ on n.

(3) Is the error $\|x - y\|$ of the order predicted by the size of roundoff error and the condition number of A?