

Sample questions and instructions for the quiz
Thursday, February 21, 30 minutes

Corrections: (none yet)

Instructions.

- You may not any materials or electronics except ...
- You may bring one sheet of paper (a cheat sheet) with anything written on it. The only requirement is that you prepare it yourself and that you can read it without assistance.
- You get 20% of the points on any question if you leave it blank.
- Anything you write that is wrong will count against you, even if you also write the correct answer. If you change your mind about an answer, cross out the part you think is wrong.
- For True/False and multiple choice questions, write a few words or a few sentences explaining your answer.
- There are more sample questions here than there will be on the actual quiz

True/False. For each statement, say whether it is true or false and explain.

1. It is impossible to replicate an asset at time T in the future if its at time T in the future depends on a price S_T that is unknown when the asset is created.
2. The forward price (for delivery at time $T > t$) of a dividend paying stock is $F_t = S_t e^{r(T-t)}$.
3. Suppose there are two assets with price $S_{1,t}$ and $S_{2,t}$. Suppose that in one period stock 1 can go up by a factor of u_1 or down by a factor d_1 , and stock 2 can go up or down by factors u_2 , or d_2 . Suppose there is an exchange option that after one period allows you to exchange one share of stock 1 for a share of stock 2. It is possible to replicate this option with a portfolio consisting of Δ_1 shares of stock 1, Δ_2 shares of stock 2, and cash.

Multiple choice

1. In the binary, two state model of hedging a call option, which two instruments are used to replicate the option?

- (a) The underlying asset, whose price is S_t at time t , and cash, which turns C at time zero to Ce^{rt} at time t .
 - (b) A zero coupon bond that matures at t and a floating rate LIBOR loan $L_{0,t}$.
 - (c) The underlying asset and the call option option.
 - (d) A put option and a call option on the same underlying asset
2. The right to take out a loan at a specified time for a specified duration at a specified interest rate is called
- (a) An interest-free loan
 - (b) A forward rate agreement (FRA)
 - (c) A zero coupon bond
 - (d) A put
3. Why (in the theory presented in the book) might a loan in currency A have a different interest rate than a loan in currency B ?
- (a) Because exchanging money from one currency to another is difficult.
 - (b) Because the price to buy one unit of B using A might be different from the inverse of the price of buying one unit of A using B (the exchange rate).
 - (c) Because the exchange rate itself can change during the duration of the loan.
 - (d) Because accounting rules in the two currencies may be different.

General questions

1. Suppose a zero coupon bond that pays \$1 (or just 1) at time t costs Z_t today. Suppose you want to agree today ($t = 0$) to swap a loan at LIBOR at time t for repayment at time T with a loan at time t to be repaid at time T (with interest) at a fixed rate R agreed on today ($t = 0$). Find a formula for R in terms of Z_t and Z_T makes this forward rate agreement worth zero today. Explain how to replicate the swap using time t and time T zero coupon bonds.
2. Suppose The $R\mathcal{E}O$ index (a fictional alternative to $S\mathcal{E}P$) has value $S_0 = \$100$ today. Suppose a forward contract to deliver one unit of this index in one year has zero value today and the delivery price is $F = \$105$. What is the risk free interest rate implied by these numbers? Do not use a calculator. Calculate the implied rate to within one basis point using a suitable number of Taylor series terms of a suitable function.
3. Suppose the force of mortality is modeled as $\mu(a) = \lambda a$, with $\lambda = .001$. What would be the probability that a person living at age 60 would die within the next six months?

4. A *capped call* is a call option with strike price K , but whose value may not exceed L (the cap). Draw the payout diagram for a capped call.
5. Write a script in R that computes the following sum using a `for` loop.

$$S = \sum_{k=1}^n e^{-at_k^2},$$
$$t_k = k/m.$$

The script should start with the commands

```
a = .02  
m = 4  
n = 20
```