Quiz Tuesday, February 26, 30 minutes

Corrections: (none yet)

Instructions.

- You may not any materials or electronics except one $8\frac{1}{2}'' \times 11''$ piece of paper with anything you want written on it.
- You get 20% of the points on any question if you leave it blank.
- Anything you write that is wrong will count against you, even if you also write the correct answer. If you change your mind about an answer, cross out the part you think is wrong.
- For True/False and multiple choice questions, write a few words or a few sentences explaining your answer.

True/False. For each statement, say whether it is true or false and explain.

1. Let Z_T be the price today of a zero coupon bond that pays \$1 at time T. If the interest rate from now to T is positive, then $0 < Z_T < 1$.
2. If there are zero coupon bonds available at any maturity time T, then a fixed term annuity may be replicated exactly with a portfolio of zero coupon bonds. Fixed term means the dates and amounts of annuity payments are specified when the annuity is purchased.
3. If you look at interest rates implied by LIBOR or US Treasuries, the interest rate is interest rate for 6 months and ten years is the same.

Multiple choice

1. Suppose a customer wants to arrange a loan to be taken at time t and repaid at time $T > t$. The customer wants a contract today that specifies the interest to be paid. Suppose interest rates fluctuate but there is no danger that the loan will not be repaid. What information is used to determine this rate?
   (a) The probability that interest rates will increase between now and time t.
   (b) The costs today of a zero coupon bonds that matures at time t and at time T, Z_t and Z_T.
   (c) The costs today of zero coupon bonds Z_T and Z_{T+t}.
   (d) The expected returns $\mu_t = \mathrm{E}[(S_t - S_0)/S_0]$ and $\mu_T = \mathrm{E}[(S_T - S_0)/S_0]$ on an easily traded asset.

General questions on the other side

General questions

1. Consider countries 1 and 2 with fixed continuously compounded interest rates (in their currencies) r_1 and r_2. Suppose that today you can buy one unit of currency 2 with N units of currency 1.

   (a) Find a formula for the forward price at time t to buy one unit of currency 2 with units of currency 1. This is the price at which the contract is worth zero today.

   (b) Explain the arbitrage that would allow you to make a risk free positive cash flow if there is a contract to do the trade with price $P > F$.

2. Suppose a portfolio consists of a European style put and a European style call on the same asset S_t with the same strike price K. Sketch the payout diagram for the cash flow from this portfolio at time t as a function of S_t.

3. Suppose $r = 4\%$ is the continuously compounded interest rate. Find the equivalent interest on a six month loan (half a year) if it is not compounded. That is $\$N \rightarrow \$N(1 + y)$ in six months with 4% continuous compounding (that is 4%/year). Use Taylor series to find y to within one basis point.

4. Consider a one period binary CRR pricing model for a call option with strike $K = S_0$. Consider the possibility that the asset price could increase or decrease by 20%, which means that $u = 1 + .2$ and $d = 1 - .2$. Suppose the risk free interest rate is $r = 0$.

   (a) Find the arbitrage price of the call if $S_0 = 100$.

   (b) Use a first derivative approximation (not the exact formula) to estimate how much more the call would be worth if u is replaced by $1 + .21 = 1 + .2 + \epsilon$ with $\epsilon = .01$.

5. Write code in `R` that computes the sum $S = \sum_{k=1}^{n} k a_k b_k$. Suppose that the numbers a_k and b_k are the first n entries in the arrays `a` and `b`. Suppose n is in `n`. Just give the initialization of `S` and a `for` loop to compute the sum.

True/False and multiple choice questions on the other side