Assignment 4.

Given December 1, due December 14.

Objective: Explore solution of Stochastic Differential Equations.

We have a portfolio of 4 American style puts, each with an expiration of 6 months. The allocation is:

stock	spot	strike	growth rate	number
1	80	80	.4	5
2	32	30	.3	10
3	30	35	.6	-15
4	50	50	.5	-5

We are not going to hedge these options (as Black and Scholes say we should), but we will exercise them (or have them exercised on us) according to the Black Scholes theory. If we exercise an option, we then receive the risk free rate (5% per year) for the remaining time. We pay the risk free rate if an option is exercised on us. We want to make a histogram of the value of our portfolio at the end of six months. Assume that stock j has price $S_j(t)$ which is a geometric Brownian motion

$$dS_j = \mu_j S_j dt + S_j dX_j \tag{1}$$

Where the X_j are correlated Brownian motions. The covariance of the X_j at time 1 (year) is

1.7000	0.8400	0.1800	0.3600
0.8400	1.9840	0.3780	0.1800
0.1800	0.3780	1.5760	0.0400
0.3600	0.1800	0.0400	0.9120

Write the system of stochastic differential equations appropriate for this problem. That will involve some kind of square root of the covariance matrix (possibly Choleski) and knowing the early exercise point for American puts (possibly from your previous finite difference computation). Solve the SDE by forward Euler. Verify that the bias is proportional to Δt. This uses lots of computer time, but a manageable amount if you choose parameters appropriately. When you get to the histogram, try to make error bars on the estimated values. Choose the bin sizes and number of paths in a sensible way and explain you choice. Compute error bars based on elementary large sample statistics. In a future assignment, you will compute sensitivities and make other modifications.