Assignment 3.

Given October 13, revised October 14, due October 27.

Objective: To do a more serious finite difference computation.

We want to value a proposed financial contract called an AI option. An American style AI option initially has two options that may be exercised at any time. One of the options is to sell a share of stock (the Asset of the Asset-Index option) at the strike price, K. Once this option has been exercised, the contract is settled and terminated. The other option is to convert from the underlying being one share of A to being an equivalent amount of the index, I. If the holder exercises this option, then he or she may, at any later time, sell an equivalent amount of the index at the strike price, K. This would result in cash flow $\max(K - S_I, 0)$. The holder may convert from asset to index only once, but need not exercise the index option immediately upon conversion.

We want to price such an option in the Black–Scholes world. For this, we assume that the asset price, S_A, and the index price, S_I, are both geometric Brownian motions with volatility parameters σ_A and σ_I respectively. The correlation coefficient between the S_A and S_I innovations (or shocks or returns) is ρ_{AI}.

1. Give the hedging argument and derive the PDE that prices AI options. Be careful with boundary conditions.

2. Discretize this PDE using a second order accurate method.

3. Find the price for $K = 100$ and spot prices at the money: $S_I = S_A = 100$. Take parameter values $\sigma_A = 1$, $\sigma_I = .2$, $\rho_{AI} = .6$, and $r = .05$, in annualized units. Take the overall expiration time to be six months. Do enough of a convergence study to be clear that the method is second order accurate and that your answer is within .1% of the correct answer.

4. Compute $\Delta_A = \partial_{S_A} f$ and $\Lambda_A = \partial_{\sigma_A} f$, again to within .1%.

5. Repeat parts 3. and 4. under the assumption that whenever $S_A < 70$, σ_A goes up to 1.5 and ρ_{AI} goes down to .2.

Hint: One of the things you have to figure out is the value of the index option. This is just a vanilla American option valuation problem that has nothing to do with the the Asset price S_A. You might want to warm up on this problem, which is a one dimensional PDE with a simple early exercise boundary.