Assignment 1.

Given September 11, due September 25.

Objective: To become familiar with the basic operations of numerical analysis.

(1) Write a program to compute

$$\int_a^b f(x)dx$$

using a panel method with a uniform panel size. The program should take as input data a, b, and the panel size. The integrand, f, is specified by writing a subroutine that returns $f(x)$ for an arbitrary value of x. Make sure the program is modular so that changing the integration rule is easy. The program should do something thoughtful in case the integration interval is not an integral multiple of the panel size.

(a) Do a convergence study to verify the second order accuracy of the trapezoid rule and the fourth order accuracy of Simpson's rule using

$$\int_0^1 \frac{1}{1+x^2}dx$$

as a test problem.

(b) What happens if you use $\int_0^1 x^3 dx$ as a test problem?

(c) What happens if you do part (a) using single precision arithmetic?

(d) Take $g(x) = 0$ if $x \leq 1/\sqrt{2}$ and $g(x) = (x - 1/\sqrt{2})^2$ if $x > 1/\sqrt{2}$. Do a convergence study to determine the accuracy of the trapezoid rule and Simpson's rule for $\int_0^2 e^{g(x)}dx$. What accounts for the discrepancy between the "theoretical" accuracy of Simpson's rule and its measured accuracy on this test problem?

(2) Write a program that takes as input an interval $[a, b]$, an error tolerance ϵ, and a function $f(x)$ (specified as a subroutine), and outputs a list of x values, x_1, x_2, $\cdots$, x_n, and corresponding f values, $f_k = f(x_k)$, so that linear interpolation from those values gives an approximation to f that is less than ϵ everywhere. Start with a small number of intervals, estimate the interpolation error on each interval, and repeatedly bisect those intervals whose estimated error is larger than ϵ. Apply your program to the function

$$f(x) = x^4 + e^{-x^2/2} - 2e^{-40x^2}$$

on the interval $[0, 2]$, with $\epsilon = .05$. Use well designed graphics to illustrate the progress of the adaptive algorithm, possibly by plotting the error after each bisection stage.