Research Problems

1. Show, in the Lagrangian setting, that Gertsner's waves (see problem 3 of Hwk. 1) actually solve the equations of motion for an inviscid fluid of constant density with gravity, and represent gravity waves with a free surface at constant pressure. For what a, b are the fluid particles on the surface of the wave?

2. In cosmology, the acretion of matter into stars, planets etc. may be studied as a problem of fluid dynamics. Assume an isothermal gas, equivalent to the barotropic gas with $p = k\rho$ for a constant k. The gas is inviscid, and subject to a gravitational field $\vec{F} = \nabla\Phi$. The key point is that gravity is due to the matter itself, according to the equation $\nabla^2\Phi = -G\rho$ where G is the gravitational constant. The problem is to study this system *in one space dimension*, using Lagrangian coordinates. Try to set up a mathematical formulation that would allow a study of gravitational accretion.

3. Investigate the particle paths of the Beltrami fields $\vec{u} = (Bsiny + C\cos z, C\sin z + A\cos x, A\sin x + B\cos y)$ of problem 8, Hwk. 3. Choose contants A, B, C, then integrate various paths. Since the field is periodic on a $2\pi^3$ cube, you can plot the intersections of any particle path with the xy, yz, or zx plane mod 2π (a *Poincaré section*). Irregular scatter of points, filling a finite area of the plane, is a signal of a chaotic Lagrangian path.