Fluid Dynamics I PROBLEM SET 7 Due November 3, 2003

1. Consider the Joukowski airfoil with $\zeta_0 = bi$ and $a > b > 0$. The circle in the ζ-plane passes through the points $(\pm a, 0)$. (a) Show that the airfoil is an arc of the circle with center at $(0, -(a^2 - b^2)i/b$ and radius $(a^2 + b^2)/b$. (b) With Kutta condition applied to the trailing edge, at what angle of attack (as a function of a, b)is the lift zero?

2. Consider a 3D wing of high aspect ratio. Let the airfoil parameters other than chord (i.e. k, β) be independent of y, the coordinate along the span of the wing. Also, assume the planform is symmetric about the line $x = 0$ in the $x - y$ plane. Using Prandtl's lifting-line theory, show that for a given lift the minimal induced drag occurs for a wing having an elliptical planform. Show in this case that the coefficient of induced drag $C_{D_i} = 2 \times drag/(\rho U^2 S)$ and lift coefficient $C_L = 2 \times lift/(\rho U^2 S)$ are related by

$$C_{D_i} = C_L^2/(\pi A).$$

Here S is the wing area and A is the aspect ratio $4b^2/S$. (Some of the WW II fighters, notably the Spitfire, adopted an approximately elliptical wing.)

3. A 3D body D moves steadily with velocity $\mathbf{U}$. The flow is a potential flow exterior of the body, phi being the potential for the flow relative to stationary fluid at infinity, and $\frac{\partial \phi}{\partial n} = \mathbf{U} \cdot \mathbf{n}$ on the body surface ∂D. Given that for large $R^2 = x^2 + y^2 + z^2$ the potential decays like

$$\phi = -\frac{a}{R} - \frac{\mathbf{A} \cdot \mathbf{R}}{R^3} + O(R^{-3}),$$

where $a, \mathbf{A}$ are constants (scalar and vector respectively), show that necessarily $a = 0$. (Note: $\int_{\partial D} \mathbf{n} \cdot \mathbf{U} dS = 0$.)

4. Justify the expression for total fluid energy exterior to the body in problem 2 above, as measured relative to the stationary fluid at infinity:

$$E = \frac{1}{2}\rho(4\pi \mathbf{A} \cdot \mathbf{u} - V_0 u^2),$$

where V_0 is the volume of the body.

5. From the results for 2D potential flow given in class, show that the apparent mass of a flat plate of length 4a moving broadside on is $4\pi a^2 \rho$.