Algorithmic Approach to Taxable Investing
Betterment

Automated, online financial advising platform

- Goal-based advice and investment management
- Higher returns at a lower cost by using technology to automate optimal investing practices
Robo-advisor

“provides financial advice or investment management online with moderate to minimal human intervention. They provide digital financial advice based on mathematical rules or algorithms, executed by software.”
Better after-tax, after-fee investing outcomes

- Low costs
- Diversified global portfolio
- Automated portfolio management
- Tax optimization
- Good investing behavior

Annual IRR over 30 year investment horizon vs US-only stock/bond portfolio
Determinism in investment management

- Low costs
- Taxes
- Trading
- Security selection
- Asset allocation

Least certain to Most certain
Taxes are a huge consideration for retail investors

- Taxes can be a 0.5% - 2.0% annual drag on returns depending on portfolio turnover and tax bracket
- Presents additional opportunities to generate outperformance for investors
- Easier than “beating the market”
Tax minimization strategies

- Tax loss harvesting
- Asset location
- Lot sorting
Tax loss harvesting

- Sell security at a loss to realize capital loss for tax purposes
- Buy correlated security to keep market exposure
Tax Loss Harvesting: An example

Emerging Markets, 1/2/2014 – 5/21/2014

$331 loss harvested

Remain in alternate ticker
How losses can be used

- Defer tax on capital gains
- Shift capital gains from short-term to long-term
  - Short-term losses against short-term gains
  - “Net the nets” -- excess long-term losses against short-term gains
- Reduce taxable income up to $3,000
- Carry excess losses forward into subsequent tax years
- Permanent tax avoidance in certain situations -- charitable donation, bequest
Setting up the problem

Maximize losses harvested within a tax year while maintaining target allocation

- Look for harvesting opportunities daily
- Determine when to harvest vs wash sale lockout period
- Avoid wash sales
Wash sales are a big constraint

The **wash sales rule** disallows a loss from selling a security if a “substantially identical” security is purchased 30 days after or before the sale.

“Substantially identical” not explicitly defined by the IRS.
Alt. tickers solve two wash sale challenges

1) Inflows
   - Allows new inflows to be allocated while avoiding the buying back securities that were purchased 30 days before

2) Switch back to primary
   - Blindly switching back to primary security after 30 days can *increase* tax liability
   - Standard tax loss harvesting implementation
30-day switchback can have neg. value

- If alternate ticker goes up, investor would realize STCG
- QDI only applies after 60 days

TLH with 30-day Switchbacks

Emerging Markets, 1/2/2014 – 5/21/2014

$360 short-term gain

$331 loss harvested
Avoiding negative tax alpha pays off

Average Annual Tax Offset by Strategy

70% stock, 2000-2013

- Tax Loss Harvesting+: 1.94%
- Other Automated TLH (with 30-day switchback): 0.95%
Choice of alternate tickers is important

Alternate security should have 1) high correlation to primary security, 2) comparable fees, and 3) sufficient liquidity.
Setting threshold for harvesting a loss

1) Expected benefit of harvest should be greater than transaction costs

2) Consider opportunity cost of harvesting now vs harvesting later
   - Cannot harvest in the asset class for 30 days because of wash sale rule
Setting threshold for harvesting a loss

1) Expected benefit of harvest should be greater than transaction costs
   - Very low volatility assets are not TLH’ed
     - Short-term treasuries
     - Short-term TIPS

2) Consider opportunity cost of harvesting now vs harvesting later
   - Cannot harvest in the asset class for 30 days because of wash sale rule
Setting harvest triggers using options theory

Harvesting a loss is similar to writing a 30-day put struck at the harvest price on the security

- We ‘lose’ when the value of the security drops below the harvest price (ie: we could have harvested at a better price)
Benefit of harvest over holdout period

Potential benefit

Price of security

Harvest price
Harvest when value harvested > option value

Harvest loss when:

Basis - Price > American put (K = price, t = 30)

- Harvested amount
- Put option sold for next 30 days

Volatility of security is biggest driver of loss amount needed to harvest
Tax offsets each year

- Provide positive tax offsets in 11 out of 13 years
- No negative tax offsets over the period, even though the portfolio was regularly rebalanced.
- The mean annual tax offset was 1.94%.
TLH is backtested to demonstrate after-tax alpha over 30 years

- Average rate taxpayer
- Average savings schedule

https://www.betterment.com/resources/research/tax-loss-harvesting-white-paper/
TLH innovations

- Stay in secondary ticker after harvest (ie, don’t switch back)
- Decision to harvest considers opportunity cost future wash sales -- conditional on volatility of asset
- Rebalance with each harvest
- Considering spousals accounts to avoid wash sales
- Different secondary ticker for IRA accounts to avoid permanent wash sale rule
  - Generally, a “washed” loss is postponed until the replacement is sold, but if the replacement is purchased in an IRA/401(k) account, the loss is permanently disallowed.
- Continuous, intraday monitoring
Asset location

Putting tax-inefficient investments (bonds) in tax-efficient accounts (IRAs)
Setting up the problem

Certain assets are tax inefficient -- bonds, intl stocks (no QDI)

Accounts have different tax treatment
- Roth IRA/401k -- Growth is tax-free, no taxes on withdrawal
- Traditional IRA/401k -- Growth is tax-free, taxed as ordinary income on withdrawal
- Taxable account -- dividends taxed each year, withdrawals taxed at capital gains rate

Allocate assets to each account to maximize after-tax expected return
s.t.
- Sum of weights across accounts equal target portfolio weights
- Account balances are preserved
# Tax treatment of security types

<table>
<thead>
<tr>
<th></th>
<th>Dividends (taxed annually)</th>
<th>Capital Gains (taxed when sold)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ordinary Rate</strong></td>
<td>Most bonds</td>
<td>Any security held for a year or less (STCG)</td>
</tr>
<tr>
<td></td>
<td>Non-QDI stocks (foreign)</td>
<td></td>
</tr>
<tr>
<td>** Preferential Rate**</td>
<td>QDI stocks (domestic and some foreign)</td>
<td>Any security held for more than a year (LTCG)</td>
</tr>
<tr>
<td><strong>No Tax</strong></td>
<td>Municipal bonds</td>
<td>Any security transferred upon death or donated to charity</td>
</tr>
</tbody>
</table>
Asset Location

Not asset located:

- Self-contained diversified portfolio in each account -- Roth, Traditional and taxable accounts

Asset located:

- Maintain overall level of diversification
- Locate tax-inefficient assets in tax-advantaged accounts
Asset location can be optimized

- Can be defined as linear programming problem
- Accounts are continuously monitored
- Go beyond rules of thumb:
  - Consider growth rate of assets
    - Dividend yield and growth rate matter
  - Consider tax drag and liquidation taxes
  - QDI ratios

Asset location as a LP problem

Setup Tax Coordinated Portfolios LP:

$$\max_x \sum_j \sum_i r_{ji}x_{ji} - \gamma (\sum_i (y_i^+ + y_i^-))$$

where

- $r$ is the after-tax return for i-asset in j-th account
- $x_{ji} > 0$ - position in i-asset in j-th account
- $y_i^+$ and $y_i^-$ parametrize positive and negative drift at the asset level (slack variables)
- $\gamma > 0$ - penalty factors

Subject to the following constraints:

$$\sum_i x_{ji} = b_j$$, where $b_j$ is balance in j-th account - balance preservation constraint

$$\sum_j \sum_i x_{ji} = \sum_j b_j$$, where $b_j$ is balance in j-th account - total balance preservation constraint

$$\sum_j \sum_i x_{ji} + \sum_i (y_i^+ + y_i^-) = w_i \sum_j b_j$$, where $w_i$ is i-th asset weight in the target asset allocation - global asset allocation constraint
Allocations become more tax efficient over time
How is this different/better?

- Need to consider annual taxes and liquidation taxes
- Need to consider QDI
- Need to consider growth of assets given different liquidation taxes
- Solves LP problem on every rebalance/cash flow.
  - Constantly working to more optimally locate assets in accounts
Asset location implementation is tested for tax alpha using tax lot monte carlo simulations

As a result, investors can get higher take-home returns from TCP than they do traditionally.

<table>
<thead>
<tr>
<th>Asset Allocation</th>
<th>Additional Tax Alpha with TCP (Annualized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50% Stocks</td>
<td>0.82%</td>
</tr>
<tr>
<td>70% Stocks</td>
<td>0.48%</td>
</tr>
<tr>
<td>90% Stocks</td>
<td>0.27%</td>
</tr>
</tbody>
</table>

https://www.betterment.com/resources/research/tax-coordinated-portfolio-white-paper/
Tax lot management
Intelligent lot sorting

- Automatically manage sales at the lot-level intelligently

- Sell lots with losses first, then least gains

<table>
<thead>
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<th>TaxMin</th>
<th>Industry Standard (FIFO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3,883</td>
<td>$0</td>
</tr>
<tr>
<td>taxes saved</td>
<td>taxes saved</td>
</tr>
</tbody>
</table>

Based on an actual customer withdrawal of $100k in Apr 2014 with tax rate of 30% LT, 50% ST. [About this data]
Lot order preference

1. Short-term losses
2. Long-term losses
3. Long-term gains
4. Short-term gains *
   ○ STCG are blocked except for withdrawals
   ○ Do not incur STCG when rebalancing

Lot sorting used whenever there is a sell -- rebalancing, withdrawal, fees
Again, losses are useful

- Net against capital gains
- Reduce taxable income up to $3,000
- Carry excess losses forward into subsequent tax years
Managing tax rate uncertainty
Portfolio construction with tax uncertainty

- **Black-Litterman**
  - Add relative after-tax performance as a view
  - Can specify level of confidence
- **Robust optimization**
  - Draw from distribution of BL posterior returns, many times
  - Optimize
  - Average across sets of weights
Introduce after-tax outperformance as view in Black-Litterman framework

- BL view matrix Q holds expected relative after-tax performance
- Uncertainty of view matrix allows us to control likelihood of tax regime
Incorporating Views

Relative tax efficiency

Views \( Q \)

Confidence level (0% to 100%)

Uncertainty of Views \( \Omega \)


\[ \Omega \] is a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view \((K \times K)\) matrix;

\[ \Pi \] is the Implied Equilibrium Return Vector \((N \times 1)\) column vector); and,

\[ Q \] is the View Vector \((K \times 1)\) column vector).
Robust optimization

We employ parametric Monte Carlo and simulate many scenarios.

Assume returns are generated by a multivariate normal distribution with mean vector given by our Black-Litterman posterior expected returns estimates and covariance.

In the end, we get 100 to 500 estimated frontiers (sets of allocations) and we average the portfolio weights for each asset across these simulations.
Most meaningful difference: taxable portfolio loads on muni bonds

Increase in municipal bond allocation in taxable portfolio
Thanks!

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Estimating Posterior Moments

\[
E[R] = \left( (\tau \Sigma)^{-1} + P' \Omega^{-1} P \right)^{-1} \left[ (\tau \Sigma)^{-1} \Pi + P' \Omega^{-1} Q \right]
\]

\[ N \sim \left( E[R], \left( (\tau \Sigma)^{-1} + \left( P' \Omega^{-1} P \right)^{-1} \right) \right) \]

- **\( E[R] \)** is the new (posterior) Combined Return Vector (\( N \times 1 \) column vector);
- **\( \tau \)** is a scalar;
- **\( \Sigma \)** is the covariance matrix of excess returns (\( N \times N \) matrix);
- **\( P \)** is a matrix that identifies the assets involved in the views (\( K \times N \) matrix or \( 1 \times N \) row vector in the special case of 1 view);