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(1) (Pre-Calc Review Set Problems # 44 and # 59) Show your work.

(a) Given $\sec\theta = 5$, and $\sin\theta < 0$, find the exact values of the following functions.

$\sin(\theta) =$ ____________, $\cos(\theta) =$ ____________, $\tan(\theta) =$ ____________,

$\cot(\theta) =$ ____________, and $\csc(\theta) =$ ____________.

(b) Use the Laws of Logarithms to rewrite the following expression in terms of simple logarithm functions like $\ln(x)$, not $\ln(xy)$, $\ln(x^2)$, or $\ln(\sqrt{x})$.

$$\ln\left(x\sqrt{y\sqrt{w\sqrt{z}}}\right)$$

(2) (a) (8 points) Do the following **sequences** converge or diverge? Circle your choice. If the sequence converges, find the exact value to which the sequence converges. You do not need to explain.

(i) $a_n = \dfrac{3^n}{2^n + 4^n}$ Diverges, or, Converges to ____________

(ii) $a_n = \ln\left(\dfrac{2n+1}{n}\right)$ Diverges, or, Converges to ____________

(iii) $a_n = 1 + e^{-2n}$ Diverges, or, Converges to ____________

(iv) $a_n = \dfrac{n^{1/3} + 1}{n^{1/2} - 1}$ Diverges, or, Converges to ____________

(b) (3 points) If a is a constant and $|a| < 1$, then the infinite series $a^2 + a^5 + a^8 + a^{11} + \dots$

____________(converges / diverges). If it converges, its sum is ____________.

Show work:

(c) (4 points) If the infinite geometric series $\dfrac{1}{a} + \dfrac{1}{a^2} + \dfrac{1}{a^3} + \dots = \dfrac{4}{5}$, then what is the sum $\dfrac{1}{a^3} + \dfrac{1}{a^5} + \dfrac{1}{a^7} + \dots$?

Answer: ____________.

Show work:

(d) (15 points) Fill in the blanks. You do not need to explain.

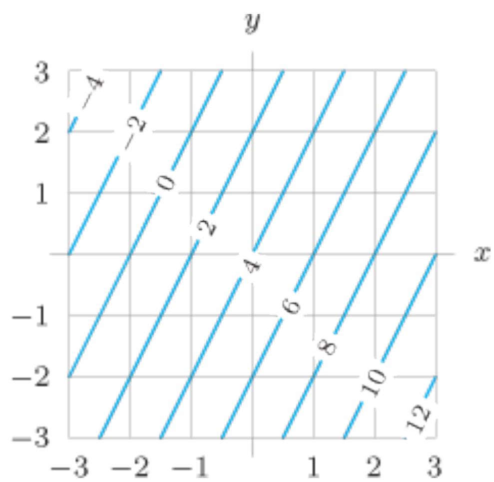
Consider three points $A = (20, 90, 50)$, $B = (-100, 0, 0)$, and $C = (50, 5, -100)$.

The one that is closest to the yz-plane is ____________.

The one that lies on the xz-plane is ____________.

The one that is farthest from the origin is ____________.

(e) Find an equation for the linear function with the following contour diagram. Show your work.



(f) Determine whether the following limit exists. If so, find it. If not, explain why it does not exist. Show work.

$$\lim_{(x,y)\to(0,0)} \frac{3x^3y^3}{x^6+3y^6}.$$

(3) Let f be a twice differentiable function. Some of the values of f and its derivatives are given in the table below.

x	$f(x)$	$f'(x)$	$f''(x)$
0	0	3	-1
1	3.5	2	-0.5
2	4	1.6	2.1

(a) Evaluate the integral $\displaystyle\int_1^2 (4x+3)f''(x)dx$.

(b) Evaluate the integral $\displaystyle\int_0^{\pi/2} 2\cos(x)f'\big(2\sin(x)\big)dx$.

(4) (a) Determine whether the improper integral

$$\int_{e^{-1}}^{e^8} \frac{1}{x\sqrt[3]{\ln x}}\,dx$$

converges or diverges. Evaluate its value if it is convergent. Show your work.

(b) Let $\mathcal{R}$ be the closed region bounded by the curve $y = 2x^2 - 1$ and the line $y = x$. Find the exact volume of the solid obtained by rotating $\mathcal{R}$ about the line $y = 3$.

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(5) (a) Find the first 4 nonzero terms of Taylor series for $y = \dfrac{1}{\sqrt{4-x}}$ about $x = 0$. Show your work.

(b) Find the first four nonzero terms in the Taylor series around $a = 0$ for the following function. (You can use the Taylor Series of the known functions.)

$$y = \frac{z^2}{\sqrt{1-z^2}}$$

(6) Determine whether each of the following series converges or diverges. Please state which test you used to make your decision.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{5 - \cos(n)}{\sqrt{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$

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Useful formulas

- *Integration by Parts*:

$$\int u\,dv = uv - \int v\,du \qquad \text{or} \qquad \int uv'\,dx = uv - \int vu'\,dx$$

- *Useful Integrals for Comparison:*

$$\int_1^\infty \frac{1}{x^p}\,dx \text{ converges for } p > 1 \text{ and diverges for } p \le 1.$$

$$\int_0^1 \frac{1}{x^p}\,dx \text{ converges for } p < 1 \text{ and diverges for } p \ge 1.$$

$$\int_0^\infty e^{-ax}\,dx \text{ converges for } a > 0.$$

- nth degree Taylor Polynomial of $f(x)$ centered at $x = a$:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

- Taylor series of $f(x)$ centered at $x = a$:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

- Taylor Series of important functions:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad \text{for } -1 < x < 1$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad \text{for } -1 < x \le 1$$

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots \qquad \text{for } -1 < x < 1$$

- Finite Geometric Series: $a + ax + ax^2 + \cdots + ax^{n-1} = \dfrac{a(1-x^n)}{1-x}$ for $|x| \neq 1$.
- Infinite Geometric Series: $a + ax + ax^2 + \cdots = \dfrac{a}{1-x}$ for $|x| < 1$.
- Ratio Test:
  For the series $\sum a_n$, suppose,
  $$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L.$$
  - If $L < 1$, then the series converges.
  - If $L > 1$, then the series diverges.
  - If $L = 1$, then the test fails.
- **Differentiation formulas**

$\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(\ln\|x\|) = \frac{1}{x}$	$\frac{d}{dx}(e^x) = e^x$ $\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$ $\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

- A formula for the distance between the points (x, y, z) and (a, b, c) in 3-space is
  $$d = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}.$$
- If a plane has slope m in the x direction, slope n in the y direction, and passes through the point (x_0, y_0, z_0), then its equation is
  $$z = z_0 + m(x - x_0) + n(y - y_0).$$
- The Lagrange error bound for $P_n(x)$: Suppose f and all its derivatives are continuous. If $P_n(x)$ is the n^{th} Taylor polynomial for $f(x)$ about a, then
  $$|E_n(x)| = |f(x) - P_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1},$$
  where $\max |f^{(n+1)}| \leq M$ on the interval between a and x.

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Here a, b, c, d are constants.

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\int x^n\ dx = \frac{1}{n+1}x^{n+1} + C,\ (n \neq -1)$
2. $\int \frac{1}{x}\ dx = \ln|x| + C$
3. $\int a^x\ dx = \frac{1}{\ln a}a^x + C$
4. $\int \ln x\ dx = x\ln x - x + C$

5. $\int \sin ax\ dx = -\frac{1}{a}\cos ax + C$
6. $\int \cos ax\ dx = \frac{1}{a}\sin ax + C$
7. $\int \tan ax\ dx = -\frac{1}{a}\ln|\cos ax| + C$

II. Products of e^x, $\cos x$, and $\sin x$

8. $\int e^{ax}\sin(bx)\ dx = \frac{1}{a^2+b^2}e^{ax}[a\sin(bx) - b\cos(bx)] + C$
9. $\int e^{ax}\cos(bx)\ dx = \frac{1}{a^2+b^2}e^{ax}[a\cos(bx) + b\sin(bx)] + C$
10. $\int \sin(ax)\sin(bx)\ dx = \frac{1}{b^2-a^2}[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)] + C, \quad a \neq b$
11. $\int \cos(ax)\cos(bx)\ dx = \frac{1}{b^2-a^2}[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)] + C, \quad a \neq b$
12. $\int \sin(ax)\cos(bx)\ dx = \frac{1}{b^2-a^2}[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x, $\cos x$, and $\sin x$

13. $\int x^n \ln x\ dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1, x > 0$
14. $\int p(x)e^{ax}\ dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \cdots + C$

    $(+ - + - + - + \ldots)$ (signs alternate)
15. $\int p(x)\sin ax\ dx = -\frac{1}{a}p(x)\cos(ax) + \frac{1}{a^2}p'(x)\sin(ax) + \frac{1}{a^3}p''(x)\cos(ax) - \cdots + C$

    $(- + + - - + + - - \ldots)$ (signs alternate in pairs)
16. $\int p(x)\cos ax\ dx = \frac{1}{a}p(x)\sin(ax) + \frac{1}{a^2}p'(x)\cos(ax) - \frac{1}{a^3}p''(x)\sin(ax) - \cdots + C$

    $(+ + - - + + - - \ldots)$ (signs alternate in pairs)

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IV. Integer Powers of $\sin x$ and $\cos x$

17. $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx,\quad n$ positive

18. $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx,\quad n$ positive

19. $\int \frac{1}{\sin^m x}\,dx = -\frac{1}{m-1}\frac{\cos x}{\sin^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2}x}\,dx,\quad m \neq 1, m$ positive

20. $\int \frac{1}{\sin x}\,dx = \frac{1}{2}\ln\left|\frac{\cos x - 1}{\cos x + 1}\right| + C$

21. $\int \frac{1}{\cos^m x}\,dx = \frac{1}{m-1}\frac{\sin x}{\cos^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2}x}\,dx,\quad m \neq 1, m$ positive

22. $\int \frac{1}{\cos x}\,dx = \frac{1}{2}\ln\left|\frac{\sin x + 1}{\sin x - 1}\right| + C$

23. $\int \sin^m x\cos^n x\,dx$:

If n is odd, let $w = \sin x$.
If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18.
If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21.
The case in which both m and n are even and negative is omitted.

V. Quadratic in the Denominator

24. $\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C,\ a \neq 0$

25. $\int \frac{bx + c}{x^2 + a^2}\,dx = \frac{b}{2}\ln|x^2 + a^2| + \frac{c}{a}\arctan\left(\frac{x}{a}\right) + C,\ a \neq 0$

26. $\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{(a-b)}(\ln|x-a| - \ln|x-b|) + C,\quad a \neq b$

27. $\int \frac{cx + d}{(x-a)(x-b)}\,dx = \frac{1}{(a-b)}[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|] + C,\quad a \neq b$

VI. Integrands involving $\sqrt{a^2+x^2}, \sqrt{a^2-x^2}, \sqrt{x^2-a^2}, a > 0$

28. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right) + C$

29. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x + \sqrt{x^2 \pm a^2}| + C$

30. $\int \sqrt{a^2 \pm x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2 \pm x^2}}\,dx\right) + C$

31. $\int \sqrt{x^2 - a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2 - a^2} - a^2\int \frac{1}{\sqrt{x^2 - a^2}}\,dx\right) + C$