

Problem Set 3

Problem 1

Exercise 3.5, page 51.

Problem 2

The purpose of this problem is to demonstrate the superiority of high order methods for ODEs in terms of their efficiency and accuracy. We take the simplest equation

$$\begin{cases} y'(t) = i \cdot k \cdot y(t), & 0 \leq t \leq 2\pi, \\ y(0) = 1, \end{cases} \quad (1)$$

and the simple method – the ν -stage ERK method of order ν , as is considered in Problem 1. What you observe in the experiment you are asked to do is applicable for more general ODEs and more sophisticated schemes.

- (a) For $k = 100$, and for each $\nu = 1, 2, 3, 4, 8, 12, 16, 20$, solve (1) with the ν -stage ERK method to 5-digit precision, namely, the relative error of solution at $t = 2\pi$ is no greater than 10^{-5} .
- (b) Find (numerically) the smallest number n of equispaced points over $[0, 2\pi]$ needed to achieve this precision for each ν . Divide the number n by k to get the (minimum) number of points per wavelength needed for the precision. Tabulate these numbers against ν .
- (c) Repeat (a) and (b) for $k = 1000$.

Problem 3

It is well-known that the IRK methods based on Gaussian points are all A-stable (see, e.g., Section 4.3, page 59–63). Write down explicitly the growth factor of the fourth order (two-stage) IRK method given on page 47 (which you tested in the previous assignment). Show that

- (a) The method is indeed A-stable.
- (b) Is the method L-stable, and why?

Problem 4

Given a simple argument to explain that there is no contradiction between the two facts.

- (i) $z = ikh$ is never inside the stability domain $|1 + z| \leq 1$ of the (forward) Euler's method (which usually means unstable solution, or explosive growth of the numerical solution).
- (ii) Euler's method converges (Theorem 1.1) for the equation $y'(t) =iky(t)$.