CS Peskin
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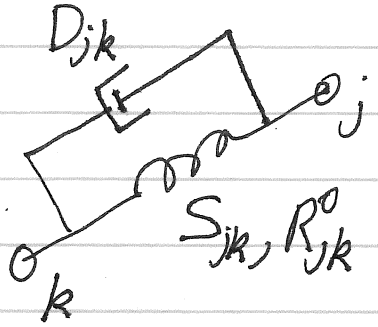
Structural Mechanics

Network of linear springs in 3D with dampers




$$M_k \frac{d\underline{U}_k}{dt} = \sum_{j \in N(k)} T_{jk} \frac{\underline{X}_j - \underline{X}_k}{\| \underline{X}_j - \underline{X}_k \|}$$

$$\frac{d\underline{X}_k}{dt} = \underline{U}_k$$

$$T_{jk} = S_{jk} \left( \| \underline{X}_j - \underline{X}_k \| - R^0_{jk} \right)$$

$$+ D_{jk} \frac{d}{dt} \| \underline{X}_j - \underline{X}_k \|$$

Evaluation of $\frac{d}{dt}\|\underline{x}_j - \underline{x}_k\|$ :

$$\|\underline{x}_j - \underline{x}_k\|^2 = (\underline{x}_j - \underline{x}_k)\cdot(\underline{x}_j - \underline{x}_k)$$

$$2\|\underline{x}_j - \underline{x}_k\| \frac{d}{dt}\|\underline{x}_j - \underline{x}_k\|$$

$$= 2(\underline{x}_j - \underline{x}_k)\cdot(\underline{v}_j - \underline{v}_k)$$

$$\frac{d}{dt}\|\underline{x}_j - \underline{x}_k\| = \frac{\underline{x}_j - \underline{x}_k}{\|\underline{x}_j - \underline{x}_k\|}\cdot(\underline{v}_j - \underline{v}_k)$$

Numerical Method

$$m_k \frac{\underline{v}_k(t+\Delta t) - \underline{v}_k(t)}{\Delta t}$$

$$= \sum_{j \in N(k)} T_{jk}(t) \frac{\underline{x}_j(t) - \underline{x}_k(t)}{\|\underline{x}_j(t) - \underline{x}_k(t)\|}$$

$$\frac{\underline{x}_k(t+\Delta t) - \underline{x}_k(t)}{\Delta t} = \underline{v}_k(t+\Delta t)$$

Note that $T_{jk}(t)$ can be evaluated from

$\underline{x}_j(t)$, $\underline{x}_k(t)$, $\underline{v}_j(t)$, $\underline{v}_k(t)$

Matlab code

Notation

$i, k$ = node indices

$l$ = link index

$jj(l), kk(l)$ = indices of nodes joined by link $l$, in either order

$$\underline{X}_k = (X(k,1), X(k,2), X(k,3))$$

$$\underline{U}_k = (U(k,1), U(k,2), U(k,3))$$

Matlab code for one time step

```
DX = X(jj,:) - X(kk,:)
DU = U(jj,:) - U(kk,:)
R = sqrt(sum(DX.^2, 2))
T = S.*(R - Rzero)
    + (D./R).* sum(DX.*DU, 2)
TR = T./R
FF = [TR, TR, TR].* DX

F = zeros(kmax, 3)
for l = 1:lmax
    F(kk(l),:) = F(kk(l),:) + FF(l,:)
    F(jj(l),:) = F(jj(l),:) - FF(l,:)
end

U = U + dt * F./[M, M, M]
X = X + dt * U
```