MJO and convectively coupled waves in a coarse resolution GCM with a simple multicloud parametrization

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Abstract

Despite recent research efforts and progress in computational power, current-operational atmospheric general circulation models (GCMs) still fail to represent adequately the dominant intra-seasonal and synoptic scale variability in the tropics, characterized by the Madden-Julian oscillation (MJO) and convectively coupled waves. Here we use the next generation NCAR GCM–HOMME, which is based on a spectral element discretization of the cubed sphere, as a dry dynamical core coupled to a simple multicloud parametrization. The coupling is performed through a judicious choice of heating vertical profiles for the three cloud types, congestus, deep, and stratiform, that characterize organized tropical convection, based on the vertical structure functions of Kasahara and Puri. Coarse resolution numerical simulations with an equivalent equatorial grid spacing of 167 km were carried out on an aquaplanet with convection restricted to the tropical band between 30°S and 30°N.

Important control parameters that affect the types of waves that emerge are the strength of the background-climatological moisture profile and the stratiform fraction in the multicloud parametrization. Here, we present results of three numerical simulations using two different background-moisture profiles and two different stratiform fractions. The first experiment uses a strong moisture background and a small stratiform fraction and provides an MJO example. It results in an intra-seasonal oscillation of zonal wavenumber two, moving eastward at a constant speed of roughly 5 m s\(^{-1}\). The second uses a weaker moisture background and a large stratiform fraction and yields convectively coupled Rossby, Kelvin and two-day waves, embedded in and interacting with each other while the last one combines the small stratiform fraction and the weak moisture background to yield a planetary scale
(wavenumber one) second baroclinic Kelvin wave. Both the intra-seasonal oscillation and the synoptic scale waves in the first and second experiments have phase speeds and zonal and vertical structures that are in excellent qualitative agreement with those of the observed MJO and convectively coupled waves, respectively. The MJO in particular has strong westerly winds trailing easterlies at the surface and has a baroclinic vertical structure while the off-equatorial flow is characterized by a quadrupole vortex. The active phase of the MJO has spikes of convection that are either standing or move in either direction, embedded in the large scale propagating envelope, featuring the zero group velocity that characterizes the MJO. Congestus heating dominates the inactive phase on the off-equatorial flanks and as such it helps precondition and restore the moisture level after the passage of a strong MJO event. The third simulation constitutes a good example of a bad MJO model that lacks some of the important features of organized tropical convective systems that are captured by the first two simulations and it implicitly demonstrates the importance of both background moisture and parametrized stratiform heating with the associated downdrafts for capturing organized tropical convection.
1 Introduction

Tropical variability is dominated by propagating cloud systems that are organized on various temporal and spatial scales ranging from meso-scale squall-line cloud clusters to synoptic-scale super-clusters to planetary-scale intraseasonal oscillations with a period of 40 to 60 days. The synoptic to planetary scale disturbances are associated with large-scale wave patterns know as convectively coupled waves and the Madden-Julian oscillation (MJO), respectively (Madden and Julian 1972; Takayabu 1994a; Takayabu 1994b; Wheeler and Kiladis 1999). As such, they are believed to play a fundamental role in regulating the weather and climate in the tropics and extratropics (Bond and Vicchi 2003; Jones et al. 2004; Zhang 2005; Straub et al. 2006) but despite the continued research efforts by the climate community, current-operational climate models represent poorly, if at all, the MJO and convectively coupled waves (Lin et al. 2006; Lin et al. 2008; Liu et al. 2009). There is a consensus in the scientific community that this deficiency is due mainly to the inadequate treatment of cumulus convection and the associated interactions across multiple temporal and spatial scales (e.g. Moncrieff and Klinker 1997; Lin et al. 2006; Majda 2007).

A new look at the cumulus parametrization problem was provided through the multicloud model of Khouider and Majda (2006a, 2006b, 2007, 2008a, 2008b; hereafter KM06a, KM06b, KM07, KM08a, KM08b, respectively), which is based on a judicious representation of the three cloud types, congestus, deep, and stratiform, that are observed to dominate tropical heating (Lin and Johnson 1996; Johnson et al. 1999; Lin et al. 2004), following previous work (Mapes 2000; Majda and Shefter 2001; Majda et al. 2004). Besides deep convection, the multicloud model includes both low-level moisture preconditioning through congestus
clouds and the direct effect of stratiform clouds including downdrafts which cool and dry the boundary layer. The multicloud model is very successful in capturing most of the Wheeler-Kiladis-Takayabu spectrum of convectively coupled waves (Takayabu 1994a; Wheeler and Kiladis 1999) in terms of linear wave theory (KM06a;KM08b; Han and Khouider 2009) and nonlinear organization of large-scale envelopes mimicking across-scale interactions of the MJO and convectively coupled waves (KM07; KM08a), in the idealized context of a simple two-baroclinic modes model. Here we propose to use the next-generation NCAR GCM, the High-Order Methods Modeling Environment (HOMME), a highly parallel code based on a spectral element discretization of the hydrostatic primitive equations on the cubed sphere (Dennis et al. 2005; Bhanot et al. 2008; Taylor et al. 2008), at coarse resolution as a dry dynamical core coupled to the multicloud parametrization.

The paper is organized as follows. In section 2, we discuss the details of the implementation of the multicloud model in the HOMME GCM by exploiting the vertical normal modes of Kasahara and Puri (1981). Section 3 is devoted to numerical simulations where three different experiments and their results are presented and analyzed. Important control parameters that affect the type of waves that emerge in these simulations are the strength of the background moisture profile and the stratiform fraction in the multicloud parametrization. The first experiment uses a strong moisture background and a small stratiform fraction and provides an MJO example. The second uses a weaker moisture background and a large stratiform fraction and yields convectively coupled Rossby, Kelvin and two-day waves, embedded in and interacting with each other while the last one combines the small stratiform fraction and the weak moisture background to yield a planetary scale second baroclinic Kelvin wave, which provides an instructive example of a bad MJO model. Finally a concluding summary
and a discussion are given in section 4.

2 The multicloud model in HOMME

One simple and efficient way to implement the multicloud model of Khouider and Majda (2006a; 2008a) in the context of an atmospheric general circulation model, with full vertical resolution, is to invoke the vertical structure normal modes of Kasahara and Puri (1981), which we denote here by $\phi_n(p), n = 0, 1, 2 \cdots$, where $p$ is the pressure coordinate. The functions $\phi_n(p), n \geq 0$ are eigenmode solutions for the Sturm-Liouville boundary value problem (Kasahara and Puri, 1981)

$$-\frac{d}{dp} \left( p^2 N^2 \frac{d \phi_n}{dp} \right) = \frac{1}{c_n^2} \phi_n,$$

with the appropriate boundary conditions at the top and bottom of the atmospheric layer $p = p_B$ and $p = p_T$. Here $N = N(p)$ is the Brunt-Väisälä buoyancy frequency and $c_n$ and $\phi_n, n \geq 0$ are the unknown eigenvalue and eigenmode solution pairs to this problem that are computed numerically using the finite-difference code of Kasahara and Puri (1981).

For each fixed $n \geq 0$, $c_n = \sqrt{gh_n}$ and $\phi_n(p)$ define the gravity-wave speed and vertical-structure function for one of the so-called vertical barotropic or baroclinic modes. Here $g$ is the acceleration due to Earth’s gravity and $h_n$ is the equivalent depth associated with the normal mode $n$ (e.g. Kiladis et al. 2009). Under the assumption of separation of variables the horizontal velocity, for instance, expands in terms of the vertical structure basis functions (Kasahara and Puri 1981; Majda 2003)

$$u(x, y, p, t) = \sum_n u_n(x, y, t) \phi_n(p) \quad (2.1)$$
where
\[
\mathbf{u}_n(x, y, t) = \langle \mathbf{u}, \phi_n \rangle \equiv \frac{1}{p_B - p_T} \int_{p_T}^{p_B} \mathbf{u}(x, y, p, t)\phi_n(p)dp, n \geq 0.
\]

Here \(x\) is longitude, \(y\) is latitude, and \(t\) is time. The vertical velocity is recovered through the incompressibility constraint,
\[
\omega(x, y, p, t) = \sum_n \omega_n(x, y, t)\psi_n(p)
\]
with
\[
\psi_n(p) = -\frac{1}{p_B - p_T} \int_{p_T}^{p} \phi_n(p')dp'
\]
and
\[
\omega_n = (p_B - p_T) \nabla \cdot (\mathbf{u}_n).
\]

Accordingly, the potential temperature expands in terms of the \(\psi_n\)'s
\[
\theta(x, y, p, t) = \sum_n \theta_n(x, y, t)\psi_n(p).
\]

As such the vertical structure functions of temperature that are associated with the first two baroclinic modes are used to define the heating profiles associated with deep convection, congestus, and stratiform cloud types. They are denoted here by \(\psi_1, \psi_2\) for simplicity in exposition. They are essentially the vertical heating profiles of multiscale organized tropical convective systems, seen in both observations (Straub and Kiladis 2002; Kiladis et al. 2005; Haertel and Kiladis 2004; Lin et al. 2004; Haertel et al. 2007; Kiladis et al. 2009; Zhang and Hagos 2009; Takayabu et al. 2009) and CRM simulations (Peters and Bretherton 2006; Tulich et al. 2006) and used in simplified models for tropical convection (see KM06a and references therein). The structure of the first two basis functions used for deep convection and congestus (shallow) and stratiform heating profiles are shown on the right panels in Figure
1(A) while the associated velocity profiles are shown on the corresponding left panels. The dashed horizontal lines, in Figure 1 (A), delimit the vertical extend of the latent heating, which is restricted to the lower atmosphere below roughly 200 hPa. The imposed total heating is given by

\[ Q_c = H_d \tilde{\psi}_1(p) + (H_c - H_s)\tilde{\psi}_2(p) \]

where \( H_d, H_c, H_s \) are respectively the deep convection, congestus, and stratiform heating rates and \( \tilde{\psi}_j, j = 1, 2 \) are the (truncated) heating basis functions equal to \( \psi_j, j = 1, 2 \) below roughly 200 hPa and zero above. While \( \psi_1 \) implies positive heating over the extent of the troposphere due to deep convection, the basis function \( \psi_2 \) implies low-level heating/cooling and high-level cooling/heating due to congestus/stratiform clouds.

The system is forced by a uniform radiative cooling \( Q_R \) on the order of 1 K/day. To guarantee an exact radiative convective equilibrium solution when the potential temperature perturbation, \( \theta \), is zero we assume that the radiative cooling has the same vertical profile as the cumulus heating:

\[ Q_R = Q_{R,1}^0 \tilde{\psi}_1(p) + Q_{R,2}^0 \tilde{\psi}_2(p). \]  

(2.4)

Note that the basis functions \( \psi_1, \psi_2 \) are normalized to unity (by rescaling the basis functions \( \phi_1, \phi_2 \)) so that the constants \( Q_{R,1}^0 \) and \( Q_{R,2}^0 \) defines the radiative cooling strength associated with the first and second baroclinic modes, respectively.

The multicloud heating and cooling are converted to a tendency in the actual temperature equation of HOMME by invoking the Exner function \( (p/ps)\kappa \) to yield

\[ \frac{DT}{Dt} = \frac{1}{1 - \kappa} M(y) \left( \frac{p}{p_0} \right)^\kappa (Q_c - Q_R) + \frac{1}{\tau_R}(T_0 - T) \]  

(2.5)

where \( \kappa \approx 2/3 \). Here \( M(y) = \frac{1}{2}(1 - \tanh(k(|y| - y_0))) \) is a smoothed Heaviside function of
latitude, $y$, to restrict the cumulus heating and cooling to the tropics. The shape of the mask is defined by setting $k = 10$ and $y_0 = 30^\circ$ and its structure is shown in Figure 1(B). The additional Newtonian relaxation term serves essentially to maintain the system near equilibrium with respect to the background temperature profile $T_0$ with a relaxation time $\tau_R = 50$ days.

In addition to the heating rates, $H_d, H_c, H_s$, the multicloud model employs a boundary layer $\theta_e$ and a vertically averaged moisture equations (KM06a; KM08a). The moisture equation is obtained by a systematic average of the moisture conservation equation written in terms of an initial sounding profile $Q(p)$ plus some perturbation $q'$ (KM06a). The advective velocity field is further truncated to the first-two baroclinic modes, that are directly excited by the imposed heating profiles, and a vertically averaged–barotropic mode (KM06a). The governing equation for the vertically averaged moisture perturbation is therefore given by,

$$\frac{\partial q}{\partial t} + \nabla \cdot (q (\bar{u} + u_1 + \tilde{\alpha} u_2)) + \tilde{Q}_1 \nabla \cdot u_1 + \tilde{Q}_2 \nabla \cdot u_2 = -P + \langle E \rangle. \quad (2.6)$$

where

$$\tilde{Q}_j = \int_{p_T}^{p_B} \frac{d\tilde{Q}(p)}{dp} \psi_j(p) dp, \quad j = 1, 2. \quad (2.7)$$

Here $\tilde{\alpha} = 0.1$ (KM06), $u_j, \quad j = 1, 2$ are the corresponding first and second baroclinic components of the horizontal velocity in (2.1), and $\bar{u}$ is the barotropic velocity vector. The truncation of the velocity field may seem somewhat arbitrary but it is done here primarily for the sake of consistency with the original two-baroclinic mode multicloud model (KM06a, KM08a) and secondly because the highly oscillating, higher baroclinic modes, do not project significantly onto the bottom heavy moisture profile. The terms $P$ and $E$ are respectively the sink and sources of water vapor in the middle of the troposphere that represent mainly
surface precipitation and evaporation of rain and detrainment of cloud water, respectively (KM06a; KM08a). The boundary layer $\theta_e$ is advected by surface wind and responds to convective and environmental downdrafts due evaporation of stratiform rain and surface fluxes,

$$\frac{\partial \theta_{eb}}{\partial t} + \mathbf{u}(x, y, p_1, t) \cdot \nabla \theta_{eb} = \frac{1}{h} E_s - \frac{1}{h} D. \quad (2.8)$$

Here $E_s$ is the evaporation from the sea surface, $D$ is the downdraft mass flux (KM06a; KM08a), $h = 500$ m is the height of the atmospheric boundary layer, which is assumed to be constant and $p_1$ is the pressure at the lowest grid level above the surface. Under the constraint of conservation of vertically integrated moist static energy, the surface precipitation and evaporation rates in the moisture equation satisfy (KM06a; KM08a)

$$P = \frac{1}{p_B - p_T} \int_{p_T}^{p_B} Q_c(x, y, p, t) dp, \quad E = \frac{D}{H},$$

where $H = 16$ km is the average height of the tropical troposphere. Note that the specific humidity, $q$, is rescaled by the ratio of the latent heat of condensation and the specific heat at constant pressure of dry air, to have units of temperature. The remaining list of multicloud variables and closure assumptions are as in KM08a. They are reported in Table 1 for the readers’ convenience and for the sake of completeness. Note that in particular the multicloud model relies on a key moisture-switch function, $\Lambda$, that depends on the dryness of the middle troposphere to allow transitions between congestus (or shallow) heating and deep convection. As mentioned earlier, the stratiform fraction, $\alpha_s$, governing the strength of stratiform heating in Table 1 is an important parameter which affects both the strength of downdrafts on the boundary layer and dynamics through (2.5). We note that the surface evaporation flux in Table 1 doesn’t include the WISHE effect, which is believed to play no
role in observations of convectively coupled waves and the MJO (e.g., Straub and Kiladis 2003).

The variables \( q, \theta_{eb}, \theta_1, \theta_2 \) in Table 1 represent deviations from a prescribed radiative-convective equilibrium (RCE) (KM06; KM08a) while the discrepancies, \( \theta_{eb} - \theta_{em} \) and \( \theta^*_{eb} - \theta_{em} \), refer to the sum of prescribed RCE values, \( \bar{\theta}_{eb} - \bar{\theta}_{em} \) and \( \bar{\theta}^*_{eb} - \bar{\theta}_{em} \), and the perturbations implied by the variables \( q, \theta_{eb}, \theta_1, \theta_2 \). Note that the RCE solution is determined by imposing the values of \( Q_{R,1}^0, \bar{\theta}_{eb} - \bar{\theta}_{em} \) and \( \bar{\theta}^*_{eb} - \bar{\theta}_{em} \) and in turn it constraints the values of second baroclinic cooling, \( Q_{R,2}^0 \), the downdraft mass flux scale, \( m_0 \), and the surface evaporation time scale, \( \tau_e \). To illustrate the effect of the stratiform fraction, the resulting cooling profile in the RCE is shown in Figure 1(C) for three different values of the stratiform fraction. Note that for \( \alpha_s = 0.5 \), the cooling maximum is shifted to the upper troposphere and reduces effective cooling to zero below roughly 800 hPa due to the stronger stratiform heating/cooling in the upper troposphere/lower troposphere. Of course, these RCE’s are unstable to waves of various types (KM06a; KM08a; KM08b; Han and Khouider 2009).

3 Numerical Simulations

Here we present the numerical results obtained by the multicloud-HOMME model for three different experiments. As pointed out in KM06, a key physical process for the large-scale convective instability in the multicloud model is large-scale moisture convergence at low level that provides an important source of moisture directly supplied by the imposed-initial moisture profile \( \tilde{Q}(p) \). Another key process is the stratiform heating that imposes both a direct cooling of the lower troposphere and an induced large-scale subsidence plus an enhancement of the downdrafts, through the evaporation of stratiform rain that cool and dry the bound-
ary layer. Both of these mechanisms are often absent in cumulus parametrizations used in contemporary GCMs (Lin et al. 2006). Moreover, the amount of stratiform heating has a direct effect on the tilt of the heating profile and thus on the large-scale waves themselves. It is therefore not surprising that variations in parameters that affect these processes will induce important changes in the model results.

The three different parameter regimes considered here are as follows. One experiment uses a stratiform fraction $\alpha_s = 0.25$ (the original value used in KM06a, KM07, KM08a, and KM08b) and moisture background constants, $\tilde{Q}_1, \tilde{Q}_2$, computed directly from the integral in (2.7) with $\tilde{Q}(p)$ inferred from the GATE (GARP Atlantic Tropical Experiment) sounding moisture profile (Grabowski 2002). In the second experiment the value of $\alpha_s$ is doubled and in the third one it is the values of $\tilde{Q}_1, \tilde{Q}_2$ that are doubled. For the reasons that shall become obvious below, we refer to the first experiment as the planetary-scale Kelvin wave parameter regime, the second as the synoptic-scale convectively coupled wave regime, and the third experiment as the MJO parameter regime and their order of presentation is reversed. The rest of the parameter values are listed in table 2 for each experiment. Note that some of the parameters such as $a_0, \alpha_c, a'_0, \gamma'_2$ deviate significantly from the standard values of KM08b, consistent with the new basis functions employed here. We discretize in space equations (2.6) and (2.8) using the spectral element methodology of HOMME and in time using Leap-Frog. The ODEs appearing in table 2 for $H_s$ and $H_c$ are integrated using a standard second order Adams-Bashforth method. The primitive equations and the multicloud equations are integrated simultaneously. The horizontal resolution is fixed to $20 \times 20$ spectral elements on each interface of the cubed sphere with each element having about three degrees of freedom. This is on average roughly equivalent to 240 grid-points or a mesh size of 167 km along the
equator, corresponding to a typical GCM coarse resolution. We use a time step of 30 seconds to resolve the convective processes.

We assume an aquaplanet with no land or topography (Grabowski 2002), where the model is set at \( t = 0 \) to a radiative-convective equilibrium. The imposed cooling is determined by \( Q_{R,1}^0 = 1 \) K and the RCE solution corresponding to \( \bar{\theta}_{eb} - \bar{\theta}_{em} = 11 \) K (a moist RCE) and \( \theta^*_{eb} - \bar{\theta}_{eb} = 10 \) K. We assume an initial-reference state of rest with a temperature profile inferred from the GATE sounding (Grabowski 2002; Grabowski and Moncrieff 2004) plus a small-random perturbation and the numerical model is evolved in time towards a statistical equilibrium for a period of 1000 to 2000 days. The horizontal root-mean-square (rms) time-series of the zonal wind in the lower and upper troposphere (at roughly 800 and 200 hPa), averaged over the equatorial belt \( 10^\circ S - 10^\circ N \), are plotted in Figure 2 for the three experiments introduced above. After a transient period of a few hundred days, the solution enters a quasi-equilibrium regime where the zonal wind rms oscillates on various time scales. The top panel which corresponds to \( \alpha_s = 0.25 \) and doubled moisture background has the longest transient period of about 800 days consisting of two stages, one between time zero and roughly 300 days followed by a relatively quiet period of about 500 days before it enters what we call the MJO regime which is characterized by intraseasonal oscillations, with an average period of roughly 80 days (about 15 peaks during 1200 days). Note that for the second transient stage, between 300 and 800 days, which is relatively stationary in terms of the zonal wind rms, the xt-Hovmöller diagrams (not shown here) are characterized by very turbulent fluctuations of fast gravity waves moving in both directions. The middle panel, corresponding to \( \alpha_s = 0.5 \) and the (original) GATE moisture background, experiences a transition period of about 600 days before it enters a regime where the top troposphere zonal wind is in a somewhat
statistically balanced state. The bottom panel, which combines $\alpha_s = 0.25$ and the GATE moisture background, has the smallest transient period of about 400 days. Note also that in the last two cases, both corresponding to the weaker (GATE) moisture background, the rms wind values do not exceed roughly 4 m s$^{-1}$ while those on the top panel, corresponding to the strong moisture background, has vigorous fluctuations reaching up to 15 m s$^{-1}$.

3.1 The MJO regime

In Figure 3, we plot the xt-contours (Hovmöller diagrams) of the meridionally averaged zonal wind in the lower and upper troposphere (left and middle panels) and the deep convection (right), which can be used as a surrogate for condensation heating and precipitation, for the experiment corresponding to the top panel in Figure 2. Planetary-scale/intraseasonal eastward moving streaks are clearly visible on each panel, occurring at an average period of about 40 to 60 days. Note the positive surface wind anomalies, reaching up to 20 m s$^{-1}$, on the left panel in phase with streaks of enhanced convection that appear on the right panel as envelopes of higher frequency/smaller scale bursts that move in all directions, featuring the zero-group velocity, reminiscent of the MJO (Wheeler and Kiladis 1999), and similar to the MJO-analog simulations of Majda et al. (2007). They are associated with streaks of easterly winds aloft, of about the same magnitude, as shown on the middle panel. This is characteristic of the Madden-Julian Oscillation where strong westerly wind bursts that are topped by easterlies aloft are seen to be in phase and slightly lagging deep convection (Kiladis et al. 2005).

We see a multitude of such wave patterns that are created, live for a while and die. Two long lived ones can be seen move in a sequence, close to each other, between times 1600 days
1800 days. One is seen to enter the domain at 0° Longitude, between times 1600 days and 1650 days, as a continuation of one that just exited on the right of the domain. It continued to travel eastward and dies near the middle of the domain at around 250° Longitude, some 40 or 50 days later. Right (about 20 days) before this demise, another streak of convection starts on the left side of the domain, between times 1650 and 1700 days, then follows at the same speed and strengthens later on, when the leading one has died. From now on we will call each such wave pattern an MJO event. The scenario described above seems to be repeated over and over. A strong MJO event is followed by a weaker one that was created a few days before the leading one dies. The weak MJO then strengthens as it propagates eastward, once the leading MJO has died. This is consistent with the TOGA-COARE data where two MJO events were seen to propagate next to each other in the same fashion described here (see Figure 3 of Yanai et al. 2000). Note also from the left and middle panels in Figure 3, the surface easterlies and high level westerlies, that mark the suppressed phase of the MJO, tend to propagate eastward at faster speeds (compared to the westerly wind burst which is clearly associated with active convection). This is consistent with the fast propagation of the MJO signals in the Eastern Pacific (Kiladis et al. 2005; Zhang 2005).

In order to pinpoint to some physical mechanisms that govern this MJO wave train, next, we focus on the two MJO events identified above between Times 1600 and 1800 days. Figure 4 displays the Hovmöller diagrams of the deep convection (left), congestus heating (middle), and vertically averaged moisture anomalies (right). As expected, the streaks of dry air are seen, on the right panel, to trail deep convection, which characterizes the active phase of the MJO. Weak but positive congestus anomalies are seen to pop up outside the active phase, after the passage of deep convection. Positive moisture anomalies are seen to precede, in
time and space, the strongest spikes of deep convection that are followed by sudden drying. The strong oscillatory interactions between moisture and deep convection is reminiscent of the MJO-skeleton model of Majda and Stechmann (2008) where the MJO is modeled as a wave train of harmonic oscillators between these two fields. The demise of the MJO deep convection is followed by periods of moistening and pre-conditioning that are dominated by congestus heating. Congestus (and in general shallow) heating is believed to help large-scale convergence of moisture at low-level (Lin and Johnson 1996; KM06a; Zhang and Hagos 2009; Takayabu 2009). In the context of the multicloud model, the congestus preconditioning operates by directly triggering second-baroclinic convergence during the suppressed phase of convectively coupled waves, in general (KM06b; KM07; KM08a). In fact, in KM06a, the instability of convectively coupled waves disappears completely and gives rise to a (non-scale selective) instability of a standing moisture mode, when the second baroclinic convergence was suppressed in the moisture equation.

The logarithm of the spectral power of the top of troposphere zonal wind and deep convection are plotted in Figure 5. A dominant peak of a planetary scale/intraseasonal eastward moving signal is evident on both plots. However, while the deep convection power is concentrated around wavenumber two and frequency 5 (a period of 40 days), the zonal wind power peak is more blurred towards low frequencies and wavenumber zero and has two apparent peaks; one at around the 40 days period and is spread horizontally between wavenumber one and wavenumber two—featuring the zero group velocity, which characterizes the MJO, and one is concentrated at wavenumber one and somewhere between 80 and 90 days. This second peak is presumably due to the fact that the two-MJO wave train has several on- and off-periods and periods where only one of the MJO’s is active as described above. Note that
the 5 m s\(^{-1}\)-dashed lines in Figures 4 correspond to a period of 45 days. Interestingly, the MJO signal is the only significant dominant peak in this simulation; synoptic-scale convectively coupled waves that are ubiquitous in satellite data, often together with the MJO signal (Takayabu 1994a, Wheeler and Kiladis 1999), are absent here. Nevertheless they dominate the spectrum of our so-called *synoptic scale convectively coupled waves* experiment reported in the next section.

To confirm the physical features of the MJO signal, we plot in Figures 6 through 8 the horizontal and vertical structure of the solution averaged along the frame moving eastward at 5.8 m s\(^{-1}\) (corresponding to the wavenumber two and 40 days period spectral peak in Figure 5), following roughly the dashed lines in Figure 4. Figure 6 displays the horizontal structure of the MJO filtered zonal wind (top two left panels), the meridional velocity (top two right panels), temperature anomalies with horizontal wind vectors overlaid (bottom left), and the relative vorticity (bottom right) at the bottom (827 hPa) and top (250 hPa) of the troposphere. A train of two wavenumber-two MJO’s is evident. In terms of the zonal winds, both MJO’s are characterized by easterlies leading westerlies at the surface and reversed flow aloft. Note that the leading MJO has a much stronger westerly winds near the surface. Unlike convectively coupled Kelvin waves, the meridional wind has significant amplitudes. Its structure suggests meridional divergence followed by convergence, at the surface, in the westerly wind burst region. Evident from the leading MJO temperature plots, a warm anomaly leads the westerly wind burst (WWB) at the surface and warm air lies over cold air within the WWB region. Both the horizontal wind vectors and the vorticity contours suggest a quadruple vortex, associated with each MJO, with a pair of cyclones/anticyclones located on both sides of the equator, followed by a pair of anticyclones/cyclones at the surface/alof,
consistent with the Gill-type response to a slowly moving heat source (Gill 1980; Biello and Majda 2005). In short the different panels in Figure 6 reflect specifically the dynamical features of the Madden-Julian Oscillation as observed in nature (Kiladis et al. 2005; Zhang 2005; Haertel et al. 2006; Kiladis et al. 2009).

The MJO filtered zonal structure of heating and moisture fields are shown in Figure 7. In agreement with observations, deep convection appears to lead and be slightly in phase with the surface westerly winds in Figure 6 while the region of surface easterlies is dominated by cooling or very little deep convective heating, consistent with the observed high-OLR suppressed phase of the MJO. The stratiform heating is in phase with deep convection and is more spatially dispersed and seems to expand the active-heating region in all directions but not necessarily significantly towards the back of the wave; this suggests a limit on the self-similarity theory of tropical convective systems (e.g. Mapes et al. 2006; Majda 2007; Kiladis et al. 2009) perhaps due to the coarse resolution.

The MJO-filtered congestus heating (bottom right panel of Figure 7) anomalies occupy the North and South flanks of the suppressed region. As such they help suck up moisture from the off-equatorial regions, where the MJO-filtered moisture field (top right) appears to peak up, towards the equator and precondition the equatorial troposphere for the next MJO event. A significant drying is associated with deep convective activity and it is more so for the first MJO event, which is followed by relatively strong congestus activity and moistening. Note also the significant negative anomaly in the boundary layer $\theta_e$ (middle right), an indication of CAPE consumption, that follows the deep convection peak and immediately peaks back up during the suppressed-preconditioning phase. Notice that the $\theta_{eb}$ anomalies, off the equator, do not seem to be affected by the MJO wave, unlike the moisture field, which responds to the
congestus heating induced large-scale moisture convergence. The peaking of both moisture and congestus heating off the equator is consistent with the persistence of equatorially non-trapped moisture and congestus modes in linear analysis of the multicloud model on an equatorial beta-plane (KM08b; Han and Khouider 2009) and is consistent with observations (Takayabu et al. 2009).

In Figure 8, we plot the vertical structure of the zonal wind, vertical velocity, temperature, and total heating, on the two top and two middle panels, respectively. We see that the two MJO’s have the expected (according to observations, e.g. Kiladis et al. 2005) baroclinic structure in terms of the zonal winds, with easterlies lying above westerlies, and vice versa, in the troposphere, below roughly 200 hPa. The temperature anomaly is characterized by warm temperatures leading and somewhat in phase with deep convection and cold anomalies trail behind and dominate the suppressed region near the surface. The warm over cold feature, in the region of active convection, is not as pronounced here unlike the case of convectively coupled waves presented below. The vertical velocity is in phase with heating with upward motions in the heating regions and downward motion in the cooling regions. The whole wave structure doesn’t seem to present a significant tilt but has more or less a first baroclinic feature (corresponding to $\phi_1$ for zonal velocity and $\psi_1$ for heating, vertical velocity, and potential temperature anomalies). The vertical structure of both zonal winds and temperature profile are more or less in agreement with the passage of the MJO over Tarawa Island, in the Pacific Ocean, reported by Kiladis et al. (2005, their Figures 4d and 8d). In fact in that work the vertical structure of the MJO varies significantly from location to location perhaps reflecting the mean climatology. Sometimes there is a significant tilt, sometimes there is not. The MJO simulated here happens to resemble more that of Tarawa
Island.

The time-zonal means of zonal wind, $q, \theta_{eb}, H_d, H_s, H_c$ that were removed from their respective MJO-filtered anomalies, in Figures 6 through 8, are reported on the bottom panels of Figure 8. We note that the mean zonal wind presents a state of equatorial super-rotation with a mean westerly wind of up to 5 m s$^{-1}$ peaking right at the equator in the middle of the troposphere between 600 hPa and 300 hPa, which corresponds roughly to the region of maximum heating as shown on the top right panel in Figure 8. Interestingly, this maximum westerly wind matches roughly the eastward propagation speed of the MJO, suggesting a steering level relationship as in squall-lines and in midlatitude synoptic waves. This is also consistent with the climatology obtained by Saravanan (1993) in a two layer model forced by a prescribed equatorial heating. Mean equatorial westerlies, i.e, equatorial super-rotation, are common in observations of the MJO (e.g. Moncrieff 2004; Zhang 2005). A theoretical study by Biello et al. (2007) suggested that the MJO’s equatorial super-rotation can be generated by planetary scale meridional fluxes, which is perhaps the case here due to the persistence of congestus heating, on the MJO scale, on both sides of the equator.

The mean deep convection, stratiform, and congestus heating rates are consistent with an RCE solution corresponding to the prescribed cooling of $Q_{R,1}^0 = 1$ K/day (KM06a) and satisfying $\bar{H}_d = Q_{R,1}^0, \bar{H}_s = \alpha_s \bar{H}_d, \bar{H}_c = \alpha_c \frac{\Delta s}{\Delta c} \bar{H}_d$, etc. Nevertheless, the mean moisture and mean $\theta_{eb}$ show substantial negative deviations, from their zero-initial states, in the equatorial region. This is consistent with the fact that the initial RCE state is taken to be moist enough to support convection in equilibrium with the imposed cooling. These negative deviations on average are therefore necessary to bring the system into a neutral equilibrium so that negative moisture anomalies will drive the system into the MJO suppressed phase.
while positive anomalies will lead to the active phase. Recall that the present model doesn’t use any grid-size cloud physics such as large-scale (stratiform) precipitation to keep the environment under-saturated everywhere in the domain and that our moisture variable is a deviation from the imposed moisture background, which defines the constants $\tilde{Q}_{1,2}$ in the moisture equation (2.6). Also outside the tropical region the convective parametrization is turned off, which explains why both the moisture and $\theta_{eb}$ deviation (from the initial RCE state) curve back to zero away from the equator. The off-tropical regions provide a somewhat artificial supply of moisture to be advected towards the equatorial region. Nevertheless, in nature such extra-tropical sources of moisture do exist; they are provided by detrainment of persistent shallow and congestus clouds on the North and South flanks of the region of deep convection, i.e, the equator (Johnson et al. 1999; Takayabu et al. 2009).

3.2 Convectively coupled waves

The Hovmöller diagrams of the top tropospheric zonal wind (left), deep convection (middle), and congestus heating (right), associated with the second experiment, corresponding to the rms time series on the middle panel of Figure 2, with $\alpha_s = 0.5$ and GATE-moisture background, are reported in Figure 9, for the last 200 days or the last 50 days. Three distinct wave patterns moving in both directions at different speeds are visible on all panels. For the longer time series, on the left, representing the zonal wind, we see both westward and eastward propagating streaks. The westward waves move as a wave packed of wavenumber four with a speed of $6.6 \text{ m s}^{-1}$ as indicated by the thick dashed line while the eastward waves that appear as smaller wavelength packets that are somewhat embedded in the larger scale westward wave-envelopes move at a much faster speed. We anticipate here that the slowly-
moving westward streaks correspond to equatorial Rossby waves while the, higher frequency and higher wavenumber, eastward streaks are actually convectively coupled Kelvin waves. The middle panel, which focuses, on the last 50 days, reveals another high frequency wave packet that moves westward at roughly 29 m s\(^{-1}\) and intersects the Kelvin waves that appear to move at roughly 18.5 m s\(^{-1}\). Interestingly, except for the left-zonal wind panel, the low-frequency, Rossby wave, streaks are hardly visible on the middle panel while the right panel displays only Kelvin wave streaks with embedded spikes corresponding roughly to the points of intersection of the dry phases of the two high frequency wave packets. It is needless to stress any further about the fact that such streaks of synoptic scale tropical cloud clusters and superclusters, that are often identified as convectively coupled equatorial waves of all sorts, are abundant in observations (e.g. Nakazawa 1988; Dunkerton and Crum 1995; Takayabu 1994b; Moncrieff and Klinker 1997; Wheeler and Kiladis 1999; Kiladis et al. 2009) and large scale cloud resolving modeling and super-parametrization simulations (Moncrieff and Grabowski 2001; Khairoutdinov and Randall 2001; Grabowski 2002; Grabowski 2003; Tulich et al. 2006) but are very seldom or non-existent in conventional AGCM’s, when they haven’t the wrong structures and the wrong phase speeds (Scinocca and McFarlane 2004; Lin et al., 2006; Lin et al., 2008).

To confirm the “identity” of the wave signals displayed in Figure 9, we plot in Figure 10 the logarithm of the spectral power of the top of tropospheric zonal wind (top) and deep convection (right). The left panels display the symmetric signals obtained by averaging between 10\(^\circ\) S and 10\(^\circ\) N, while the antisymmetric parts, obtained by subtracting the Southern hemisphere average from the Northern one, are plotted on the corresponding right panels. Apparent spectral peaks are seen, for both fields, to coincide with the dry-equatorial
waves dispersion relations, that are overlaid on top. They are indicated by the symbols (R), (K), and (WIG), for Rossby, Kelvin, and westward inertio-gravity waves, respectively. We note in particular that the Rossby wave signal is more dominant in the wind power while the Kelvin and westward inertio-gravity (both symmetric and anti-symmetric) waves dominate the power of deep convection. Consistent with observations of synoptic-scale convectively coupled tropical waves, those signals appear somewhat in between the second and third baroclinic-mode dispersion-relation curves (e.g. Wheeler and Kiladis 1999). Other (low-frequency) signals, though significantly weaker, are also visible, such as the anti-symmetric Rossby signal in the wind power plot and the somewhat lower-frequency Kelvin-like signal, elongated at a constant slope towards smaller wavenumbers, that are seen on the wind and convection power plots. Similar elongated and slow moving Kelvin signals are common in idealized GCM simulations (Frierson 2007; Kim et al. 2008). There is also a peak on the left-side of the wind power, between the (R) and the (WIG) peaks, that might be associated with easterly waves (Wheeler and Kiladis 1999) but here we will focus mainly on the three wave signals that are visible on the Hovmöller diagrams in Figure 9. Note that consistent with the Hovmöller diagrams, the spectral power plots display an average wavenumber four and a period of roughly 17 days for the Rossby waves, six and four days for the Kelvin signal, and eight and two days for the symmetric WIG signal. The antisymmetric WIG waves have a larger wavenumber, 12, and smaller period, but they appear, on a Hovmöller diagram (not shown here) to move at the same speed as (parallel to) their symmetric counterparts suggesting composite waves propagating in a non-trivial base state.

In Figures 11 through 14, we plot the physical and dynamical structure of the filtered n=1 Rossby, Kelvin, and n=1 WIG waves, respectively. The individual waves were separated
from the global solution by “time”-averaging in the frame moving along the corresponding dashed lines in Figure 9. Note the Rossby wave composite uses the 200 days time series from 1800 to 2000 days, that of the Kelvin wave is based on the 50 days between 1950 and 2000 days, while for the WIG waves the averaging period is reduced to 5 days between 1990 and 1995 days. The Rossby and the WIG signals are further separated from their antisymmetric counterparts by a symmetrization procedure.

Figure 11 displays the Rossby-filtered horizontal structure of temperature anomalies at the bottom and top of troposphere, on the top and bottom left panels, respectively, with the corresponding horizontal wind vectors overlaid (the background mean zonal wind is reported on the bottom of Figure 13). Note the familiar text-book Rossby gyres, characterized by a pair of cyclones followed by a pair of anticyclones (Gill 1982; Majda 2003; Kiladis et al. 2009). Near the surface, the cyclones are preceded by warm-temperature anomalies, on both sides of the zonally divergent hyperbolic-point at the equator while the anti-cyclones have cold temperature in front as the Rossby wave moves westward—to the left. The upper tropospheric structure is somewhat similar, hinting to some kind of a mixed first (mostly for the wind) and second (for temperature) baroclinic structure but with the temperature anomalies slightly shifted forward—to the left. The mixed first and second baroclinic structure for the zonal wind and temperature anomalies is in fact confirmed by the vertical structure plots on the right panels. As anticipated we have warm temperatures preceding the surface zonal convergence region where warm above cold temperatures prevail and the temperature anomalies above roughly 500 hPa present a front-to-rear tilt, consistent with observations (Kiladis et al. 2009, their Figure 18). As it can be seen from the spectral power peaks, the Rossby waves carry very little convective and moisture anomalies, however, contour plots (not shown here)
show deep convection in phase with warm temperatures in the upper troposphere and are preceded by positive moisture anomalies while congestus heating anomalies dominate the inactive phase within the dry regions following deep convection.

The Kelvin-averaged anomalous horizontal and vertical structures are shown in Figures 12 and 13. The near-surface zonal velocity on the top-left of Figure 12 displays double maxima almost symmetrically distributed across the equator. This off-equatorial shifting and splitting is perhaps due to interactions with the anti-symmetric Rossby and two-day waves. Otherwise, the horizontal and vertical structure displayed in Figures 12 and 13, respectively, resemble closely those of the observational records (Straub and Kiladis 2002; Kiladis et al. 2009) consistent with the linear and non-linear results of the coarse vertical resolution multicloud model (KM06a; KM07; KM08a; KM08b). Deep convection is sandwiched between low-level westerlies and easterlies and somewhat extended toward the westerly phase in a way that is less pronounced than in the MJO case. The zonal velocity in the upper troposphere has an opposite sign suggesting a baroclinic nature but easterlies over westerlies are almost in quadrature, consistent with the pronounced front-to-rear tilt, which is evident in Figure 13, as the wave moves eastward—to the right. Surface warm temperatures, on the top-right panel in Figure 12, precede and almost in quadrature with the region of deep convection where surface cold temperatures prevail. Note also the chronological sequence starting with a peaking of the boundary layer $\theta_e$ (bottom right of Figure 12) followed by a positive moisture anomaly (just above), which triggers deep convection. This is then followed by congestus heating that starts the preconditioning process, in a way that is similar to the MJO case discussed above. Note that congestus heating peaks up as soon as the boundary layer $\theta_e$ (i.e CAPE) starts to build up, after the stratiform downdraft phase that
follows deep convection ceases, while the middle troposphere is still drying. Clearly here congestus is not the dominant moistening mechanism as the atmosphere continues to dry out even after the passage of congestus heating. The vertical structure plots in Figure 13 display a pronounced front-to-rear tilt in zonal wind (top left) and a somewhat similar tilt in temperatures above 500 hPa. Note also the warm-over-cold temperatures characterizing the region of deep convection. The vertical velocity profile, on the top right panel, shows upward motion in phase with deep convection, characterized by progressive deepening from east to west, starting by rising air in the lower troposphere only and ending by upper tropospheric lifting, perhaps due to stratiform heating (not shown here) that lags deep convection by a few degrees. However, this front-to-rear tilt is not reflected in the heating profile on the bottom-right panel. Therefore, it is sensible to conclude here that this filtered-Kelvin solution is not a simple linear response to the apparent heating; in other words this suggests that the displayed structure is heavily influenced by complex interactions between the different synoptic scale waves present in this experiment. We give evidence for this next.

The zonal and vertical structures of the two-day waves are shown in Figure 14. The temperature and horizontal velocity arrows on the two top-left panels confirm again the text-book structure of two-day waves (Haertel and Kiladis 2004; Haertel et al. 2008) consistent with the linear results in KM08b. The flow patterns, of each individual wave, are characterized by a two-directional convergent point followed by a similarly divergent point with surface warm temperature anomalies preceding the convergent flow while cold temperature anomalies prevail at the rear half of the convergent zone, as the wave propagates westward (to the left). All the other features including the vertical structure of temperature and zonal wind are similar to those of the Kelvin waves in Figures 12 and 13 and to
some extent the MJO, modulo the direction of propagation of the wave, with some subtle exceptions, in agreement with the self-similarity of synoptic-scale convectively coupled waves (Mapes et al. 2006; Kiladis et al. 2009). Nevertheless, one of the most striking differences, when compared to the Kelvin waves in Figures 13, resides in the heating and vertical velocity vertical profiles on the two bottom-right panels of Figure 14. In the latter, we see a significant front-to rear tilt in the heating profile, which is consistent with the tilting of the vertical velocity. This suggests that the two-day waves constitute the primary-small scale linear response to the induced heating while the Kelvin and Rossby waves are larger-scale envelopes that are forced significantly through residual mean heating and upscale fluxes by the embedded two-day waves, which play here the role of squall-lines, in such a coarse resolution simulation (Grabowski and Moncrieff 2001; Biello and Majda 2005; Majda 2007).

We conclude this section by stressing that this “convectively coupled wave” regime has a super-rotation mean state (bottom-left of Figure 13) that is similar to the MJO case in Figure 8, with the same maximum mid-level westerly jet along the equator of about 6 m s$^{-1}$ but peaks at a lower altitude, at about 700 hPa, and has a weaker surface easterly wind of about -2 m s$^{-1}$. The mean heating and moist thermodynamic fields on the bottom-right panel are also almost identical to those of the MJO case with the same drying of both the middle troposphere and boundary layer. However, unlike the MJO case, with much stronger anomalies in the MJO wave, here the anomalies and mean winds have comparable strength as indicated in Figure 2.
3.3 A fast-moving planetary-scale Kelvin wave or a fake MJO

The Hovmöller diagrams of high-level zonal wind, deep convection, and congestus heating, corresponding to the third simulation on the bottom of Figure 2, are shown in Figure 15. A planetary scale eastward moving wave that continuously circulates the globe at a speed of roughly 29 m s$^{-1}$, the speed of the second baroclinic Kelvin wave, is apparent on each panel.

The physical structure of the filtered-wave, following the 29 m s$^{-1}$-dashed line in Figure 15 for the last 50 days, are shown in Figure 16. The horizontal contour plots, on the right-top panels, suggest a Kelvin-like structure that is at odds with observations and the typical synoptic scale waves of the multi-cloud model (KM06a; KM07; KM08a; KM08b) unlike the ones in the previous simulations. Perhaps the oddest feature is the misalignment of the temperature anomalies with respect to the convection center. Warm anomalies at the surface are in phase with the heating-westerly phase while the onset-easterly phase is cold and cold air sits on top of warm air within the active region. Interestingly, the congestus heating is relatively large; it is more than 50% of the deep convection. Consistently, the vertical structure plots on the bottom four panels, show a zonal wind with a strong mid-level jet and a temperature profile that projects heavily on the second baroclinic (compare to $\psi_2$ in Figure 1) and therefore the low-level warming, in the region of deep convection, is arguably due to the significant congestus heating that shifts the total heating to low-levels. Note also that the vertical heating profile is tilted backwards (i.e. upwards from rear to front) due to the slight lagging of congestus heating with respect to deep convection. The vertical velocity exhibits oscillations due to the 13 m s$^{-1}$ synoptic waves seen on the deep convection Hovmöller diagram, that are not completely filtered out by the relatively
short averaging period. Longer-filters yield less oscillations in the vertical velocity but they blur and weaken significantly the other fields. We conclude this section by noting that the time-zonal mean zonal wind for this experiment (not shown here) has a typical trade-wind climatology characterized by weak equatorial eastwesterlies in the troposphere below 200 hPa without superrotation, unlike the first two simulations, perhaps due to the weaker deep convective heating.

4 Concluding summary and discussion

Numerical simulations of organization tropical convective systems using the High-Order Methods Modeling Environment (HOMME), as a dry dynamical core (Dennis et al. 2005; Bhanot et al. 2008; Taylor et al. 2008) with coarse resolution coupled to the multicloud convective parametrization of Khouider and Majda (KM06a; KM08a) are presented here for the idealized case of an aquaplanet with the convective forcing restricted to the tropical region between roughly 30°S and 30°N. This multicloud model parametrization has two features absent in many contemporary GCMs (Lin et al. 2006): low-level moisture preconditioning through congestus clouds and the direct effect of stratiform clouds including downdrafts which cool and dry the boundary layer. Three distinct experiments are considered. One with a relatively moist environment background and a moderate stratiform fraction, one with a drier background and large stratiform fraction, and the third one combines the dry environment and the small stratiform fraction. The first two experiments demonstrate considerable skill in this coarse resolution GCM to produce an MJO and convectively coupled waves with many features of the observational records.

The moist experiment yields persistently intermittent MJO events that move eastward
at 5 to 6 m s$^{-1}$ of wavenumber two and a period of roughly 40 days, as envelopes of very short-lived convective bursts that are either standing or move in either direction. The MJO signal is evident in both the Hovmöller diagrams and the spectral power plots of all the convective and dynamical fields (e.g Figures 4,5,6). The physical and dynamical structure of the MJO-filtered anomalies, shown in Figures 7 and 8, resemble closely those of the observations reported in Kiladis et al. (2005), especially at Tarawa station. Some of these features include a mostly first baroclinic vertical structure of both temperature and zonal winds, a strong low-level westerly wind burst in phase and slightly lagging deep convection, and off-equatorial quadruple vortices. Surface easterlies dominate the suppressed-dry phase characterized by congestus (low-level) heating on the North and South flanks of the equator, as in the observations of Johnson et al. (1999) and Takayabu et al. (2009), that help converge moisture towards the tropics, consistent with the persistence of moisture and congestus modes, in dry and/or strong shear environment, in the linear analysis of KM08b and Han and Khouider (2009). The strong correlation at large scales of deep convection and moisture variability is reminiscent of the MJO-skeleton model of Majda and Stechmann (2009) that features the MJO as a large scale harmonic oscillator wave train between precipitation and low-level moisture.

The large stratiform fraction case yields convectively coupled Rossby, Kelvin, and westward inertio-gravity (WIG) wave packets that move respectively at $-6.6$ m s$^{-1}$, 18.5 m s$^{-1}$, and $-29$ m s$^{-1}$ as in observations of synoptic scale tropical superclusters (e.g. Kiladis et al. 2009). Interestingly the Rossby and Kelvin waves have synoptic scales corresponding roughly to wavenumbers four and six, respectively, while the WIG waves have a larger wavenumber eight and appear to be the precursors that carry most of the deep convection power em-
bedded in both the Kelvin and Rossby wave envelopes. Interestingly, most of the congestus heating is carried by Kelvin waves together with a significant portion of deep convection. The Rossby waves on the other hand have a strong wind signal and a very weak convective trace. The horizontal and vertical structure of the Rossby-, Kelvin-, and WIG-anomalies resemble strongly those of observations as in KM08a and Han and Khouider (2009). However the famous front-to-rear vertical tilt is apparent in almost all fields and all waves but non evident in the heating fields of Kelvin and Rossby waves. This feature supports the idea that both Kelvin and Rossby waves, in this simulation, are not the primary-linear response to the implied convective heating but the smaller-scale two-day waves are and they probably force significant features in the larger-scale waves through multi-scale interactions (Biello and Majda 2005; Majda 2007).

The last experiment that combines the dry background with the smaller stratiform fraction leads to a single planetary-scale second baroclinic wave that circulates the globe at roughly 29 m s\(^{-1}\), corresponding to the speed of the second baroclinic dry Kelvin wave. Interestingly its dynamical and physical features are not consistent with observations; it is dominated by low-level heating that implies warm temperature below cold temperature anomalies within the active-convective region. Because of its striking planetary-scale nature (wavenumber one) and its persisting convective coupling it provides an excellent example of a bad MJO model that has the wrong dynamical structure and a too fast phase speed, as seen in many general circulation models with inadequate convective parametrizations (e.g. Chao and Chen, 2001; Lin et al., 2006; Lin et al., 2008) independent of the dry-core numerics (Kim et al. 2008). Failure to reproduce the important features of organized tropical convective systems, that are otherwise captured by the first two-simulations, implicitly demonstrates
the importance of both moisture and stratiform heating for synoptic and intra-seasonal scale organized tropical convection consistent with the findings of Lin et al. (2004).

The time-zonal mean zonal wind of the MJO experiment presents a state of superrotation with the maximum mean westerly wind of about 5 m s\(^{-1}\) in the mid-to-upper troposphere, at roughly 400 hPa, corresponding to the maximum MJO-heating. This suggests an interpretation of the MJO as a Gill response to a heating source moving at the steering level velocity of the background wind, as in the MJO model of Biello and Majda (2005) and Biello et al. (2007). In the MJO case, the anomalous winds are much stronger than the mean winds but they are comparable in the case with convectively coupled waves (Figure 2). Thus, the mean zonal wind of the convectively-coupled waves seems to present a similar superrotation state, but the associated westerly jet does not “produce” an MJO-scale eastward propagating wave perhaps because the maximum velocity is reached down below—not in the upper troposphere where the heating is maximum. Moreover, the Kelvin waves move at a much faster speed than the maximum westerly mean wind and the Rossby and two-day waves move in the opposite direction. This rises the question of the chicken-egg problem between the mean state and the MJO-disturbance: which one produces the other? However, the mean-zonal wind in the second case may provide a plausible explanation of why this simulation actually supports Rossby waves due to shear instability. Recall that in a shear-less environment, the multicloud model doesn’t support a Rossby or MRG wave instability as shown in KM08b (only the Kelvin and the few first gravity waves, including the eastward Yanai wave, can be destabilized) but when a meridional shear/barotropic zonal-wind was added both the Rossby and MRG waves can become unstable at large-scales (Han and Khourider 2009). However, the meridional and vertical (barotropic) structure of the background wind used in Han and
Khouider (2009) is substantially different from the one in Figure 13 and therefore close investigation is required in order to understand better how Rossby waves are initiated and sustained in this simulation. This will be the subject of future research.

The explicit use here of first and second baroclinic normal modes to define the heating profile and extreme sensitivity of the results to the background moisture, are two shortcomings for the development of the multicloud model into a fully reliable and independent cumulus parametrization to use in operational climate predictions. Research efforts to build a similar multicloud parametrization for tropical convection that circumvents these two limitations and avoids the excessive use of tunable parameters, is currently under investigation by the authors and will be published elsewhere in the near future.

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References


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</table>
Hovmöller (time-zonal contour) diagrams of top of troposphere zonal wind (left), deep convection (middle), and congestus heating (right), averaged in the meridional direction between 10° S and 10° N, for the second experiment with a large stratiform fraction, $\alpha_s = 0.5$ and a GATE-moisture profile. Note the difference in time range between the left and the two right panels, that represent the last 200 days and last 50 days, respectively. The dashed lines indicate wave patterns moving at speeds of $-6.6$ m s$^{-1}$ (Rossby), $18.5$ m s$^{-1}$ (Kelvin), and $-29$ m s$^{-1}$ (n=1 WIG).

Log of the spectral power in the zonal-wavenumber and frequency domain. 250 hPa Zonal velocity (top) and deep convection (bottom). The left panels show the symmetric part of the wave signal corresponding the meridional average between 10° S and 10° N while the right panels display the anti-symmetric part (Northern hemisphere, [0,10°], average minus the Southern, [-10°,0]). The dry-equatorial waves dispersion relations of the Kelvin, Yanai, and first (symmetric) and second (antisymmetric) Rossby and gravity waves are overlaid on the corresponding panels, for the four equivalent depths corresponding to the first four baroclinic modes. The strongest spectral peaks, corresponding to the wave signals form Figure 9, are highlighted: (R) Rossby, (K) Kelvin, (WIG) westward-inertio-gravity waves.

Zonal and vertical structure of the Rossby waves, focusing on a portion of the wave-packet consisting of the two strongest waves. Left: Lower (top) and upper (bottom) tropospheric temperature (filled contours) and horizontal velocity profile (arrows). Note the largest wind vector is about 0.3 m s$^{-1}$ and 0.8 m s$^{-1}$ in the lower and upper troposphere, respectively. Right: Zonal velocity (top) and temperature anomalies (bottom).

Zonal structure of the Kelvin waves, focusing on a portion of the wave-packet consisting of only two individual waves. Left (from top to bottom): Lower troposphere zonal wind, upper troposphere zonal wind, deep convection, and congestus heating. Right: Low-level temperature, high-level temperature, moisture, and boundary layer $\theta_e$. The horizontal velocity vectors are overlaid on the temperature filled contours.

Vertical structure of the Kelvin waves in Figure 12. Left: Zonal velocity (top) and temperature anomalies (middle). Right: Vertical velocity (top) and heating and cooling anomalies (bottom). Bottom: time-zonal mean zonal wind (left) and heating and moisture fields (right).

Zonal and vertical structure of the n=1 WIG waves, focusing on a portion of the wave-packet consisting of two individual waves. Left (from top to bottom): Lower troposphere temperature, upper troposphere temperature, deep connection, and congestus heating. The horizontal velocity arrows are overlaid on the temperature anomaly filled contours. Right: Vertical structure of the zonal velocity, temperature anomalies, vertical velocity, and heating and cooling anomalies.
Hovmöller diagrams of top troposphere zonal wind (left), deep convection (middle), and congestus heating (right), in the time and zonal directions, averaged between 10°S and 10°N. Note the difference in time scales between the left panel and the two others. The thick dashed lines point to the planetary-scale streaks circulating the globe at a speed of about 29 m s$^{-1}$, visible on all three panels, and the somewhat synoptic streaks of deep convection that are embedded within the planetary scale “wave-envelope” moving at an average speed of about 13 m s$^{-1}$. 

Physical and dynamical structure of the planetary-scale fast-moving Kelvin wave. Left (from top to bottom): Horizontal structure of zonal velocity at 827 and 250 hPa, deep convection and congestus heating, and vertical structure of zonal wind and temperature anomalies. Right: Horizontal structure of temperature at 827 and 250 hPa, moisture, and boundary layer $\theta_e$ and vertical structure of vertical velocity and total heating anomaly.
Figure 1: (A) Basis functions of vertical structure. Red areas on the $\psi_1, \psi_2$ plots show regions of convective heating associated respectively with the deep convection and congestus cloud-types respectively, while the blue area designate cooling aloft. The profile of the stratiform forcing is flipped around with a minus sign so that we have cooling below (red area in $\psi_2$) and warming aloft (blue area in the $\psi_2$ plot). The shear and middle jet-shear flows associated with $\phi_1, \phi_2$ are indicated by arrows. The pressure levels where $\tilde{\psi}_1, \tilde{\psi}_2$ are truncated are shown with the horizontal dashed lines. (B) Meridional structure of the mask that limits convective heating to the tropics. (C) Vertical profile of the imposed radiative cooling for three different values of the stratiform fraction $\alpha_s$. 
Figure 2: Time series of the (horizontal) root mean square of the zonal velocity in the lower and upper troposphere, averaged over the equatorial belt 10° S-10° N, for the three experiments corresponding to the MJO (top), synoptic scale convectively coupled waves (middle), and the planetary scale-fast moving Kelvin wave (bottom). See text and Table 2 for details.
Figure 3: Hovmöller diagrams of zonal wind at 788 hPa (left) and at 211 hPa (middle) and deep convection (right) averaged between 10° S and 10° N, for the last 500 days of the simulation. MJO-regime with $\alpha_s = 0.25$ and doubled-GATE moisture background profile.
Figure 4: Same as Figure 3 but for the shorter period of time, between 1600 days and 1800 days. Left: Deep convection. Middle: Congestus heating. Right: Moisture. The thick dashed lines mark a train of two (wavenumber-two) MJO events moving together at roughly 5 m s\(^{-1}\).
Figure 5: Logarithm of the spectrum power of top troposphere zonal wind (left) and deep convection (right) corresponding to the Hovmöller data in Figure 4, in the zonal wavenumber-frequency domain, showing the evidence of a planetary scale/intraseasonal spectral peak on the right side of each panel. The vertical dashed line marks the boundary between eastward and westward moving signals and the corresponding periods are labeled on the right side of the left panel and are highlighted by the horizontal dashed lines.
Figure 6: Horizontal structure of the MJO-filtered zonal wind (top left), Meridional wind (top right), Temperature with horizontal velocity vectors overlaid (bottom left), and vorticity (bottom right) in both the lower and upper troposphere.
Figure 7: Same as Figure 6 but for the deep convection, stratiform, and congestus heating rates (left) and moisture and boundary layer $\theta_e$ (right).
Figure 8: Vertical structure of the MJO-filtered zonal wind and Temperature anomalies (top left), vertical velocity and total heating (top right), time-zonal mean of the zonal wind (bottom left), and time-zonal mean deep convection, congestus, stratiform, moisture, and boundary layer $\theta_e$ (bottom right).
Figure 9: Hovmöller (time-zonal contour) diagrams of top of troposphere zonal wind (left), deep convection (middle), and congestus heating (right), averaged in the meridional direction between 10° S and 10° N, for the second experiment with a large stratiform fraction, $\alpha_s = 0.5$ and a GATE-moisture profile. Note the difference in time range between the left and the two right panels, that represent the last 200 days and last 50 days, respectively. The dashed lines indicate wave patterns moving at speeds of -6.6 m s$^{-1}$ (Rossby), 18.5 m s$^{-1}$ (Kelvin), and -29 m s$^{-1}$ (n=1 WIG).
Figure 10: Log of the spectral power in the zonal-wavenumber and frequency domain. 250 hPa Zonal velocity (top) and deep convection (bottom). The left panels show the symmetric part of the wave signal corresponding the meridional average between 10° S and 10° N while the right panels display the anti-symmetric part (Northern hemisphere, [0,10°], average minus the Southern, [-10°,0]). The dry-equatorial waves dispersion relations of the Kelvin, Yanai, and first (symmetric) and second (antisymmetric) Rossby and gravity waves are overlaid on the corresponding panels, for the four equivalent depths corresponding to the first four baroclinic modes. The strongest spectral peaks, corresponding to the wave signals form Figure 9, are highlighted: (R) Rossby, (K) Kelvin, (WIG) westward-inertio-gravity waves.
Figure 11: Zonal and vertical structure of the Rossby waves, focusing on a portion of the wave-packet consisting of the two strongest waves. Left: Lower (top) and upper (bottom) tropospheric temperature (filled contours) and horizontal velocity profile (arrows). Note the largest wind vector is about 0.3 m s$^{-1}$ and 0.8 m s$^{-1}$ in the lower and upper troposphere, respectively. Right: Zonal velocity (top) and temperature anomalies (bottom).
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Figure 13: Vertical structure of the Kelvin waves in Figure 12. Left: Zonal velocity (top) and temperature anomalies (middle). Right: Vertical velocity (top) and heating and cooling anomalies (bottom). Bottom: time-zonal mean zonal wind (left) and heating and moisture fields (right).
Figure 14: Zonal and vertical structure of the n=1 WIG waves, focusing on a portion of the wave-packet consisting of two individual waves. Left (from top to bottom): Lower troposphere temperature, upper troposphere temperature, deep connection, and congestus heating. The horizontal velocity arrows are overlaid on the temperature anomaly filled contours. Right: Vertical structure of the zonal velocity, temperature anomalies, vertical velocity, and heating and cooling anomalies.
Figure 15: Hovmöller diagrams of top troposphere zonal wind (left), deep convection (middle), and congestus heating (right), in the time and zonal directions, averaged between 10°S and 10°N. Note the difference in time scales between the left panel and the two others. The thick dashed lines point to the planetary-scale streaks circulating the globe at a speed of about 29 m s\(^{-1}\), visible on all three panels, and the somewhat synoptic streaks of deep convection that are embedded within the planetary scale “wave-envelope” moving at an average speed of about 13 m s\(^{-1}\).
Figure 16: Physical and dynamical structure of the planetary-scale fast-moving Kelvin wave. Left (from top to bottom): Horizontal structure of zonal velocity at 827 and 250 hPa, deep convection and congestus heating, and vertical structure of zonal wind and temperature anomalies. Right: Horizontal structure of temperature at 827 and 250 hPa, moisture, and boundary layer $\theta_e$ and vertical structure of vertical velocity and total heating anomaly.
List of Tables

1 List of multicloud variables and closure equations. Here $\langle \psi_1 \rangle$ denotes the vertical average of the first heating basis function while $\alpha_2, a_1, a_2, a_0, \gamma_2, a'_0, \gamma'_2, \mu, \alpha_s, \alpha_c, \tau_s, \tau_c, \tau_{\text{conv}}$ are constant parameters whose values employed here are listed in Table 2 for the three different experiments. .................................................. 62

2 List of multicloud parameters and simulations. Note the values of $\tilde{Q}_j, j = 1, 2$ in parentheses correspond to a normalization with the $L_2$-norm (rms) of the basis functions $\phi_j, j = 1, 2$ that enter in the projections of the horizontal velocity field, used in the advection term of the moisture equation. ................. 63
Table 1: List of multicloud variables and closure equations. Here $\langle \psi_1 \rangle$ denotes the vertical average of the first heating basis function while $\alpha_2, a_1, a_2, a_0, \gamma_2, a'_0, \gamma'_2, \mu, \alpha_s, \alpha_c, \tau_s, \tau_c, \tau_{\text{conv}}$ are constant parameters whose values employed here are listed in Table 2 for the three different experiments.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Equation</th>
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</thead>
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<tr>
<td>$\theta_{em}$</td>
<td>Middle tropospheric $\theta_e$</td>
<td>$\theta_{em} = q + \langle \psi_1 \rangle(\theta_1 + \alpha_2 \theta_2)$</td>
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</table>
| $\Lambda$ | Moisture switch function | $\Lambda = 1$ if $\theta_{eb} - \theta_{em} > 20$ K  
$\Lambda = 0$ if $\theta_{eb} - \theta_{em} < 10$ K  
Linear and continuous for $10 \leq \theta_{eb} - \theta_{em} \leq 20$ K |
| $H_d$ | Deep convection | $H_d = (1 - \Lambda) \left( Q + \frac{1}{\tau_{\text{conv}}} (a_1 \theta_{eb} + a_2 q + a_0 (\theta_1 + \gamma_2 \theta_2)) \right)^+$ |
| $H_c$ | Congestus heating | $\frac{\partial H_c}{\partial t} = \frac{1}{\tau_c} \left( \Lambda \alpha_c \left( Q + \frac{1}{\tau_{\text{conv}}} (\theta_{eb} + a'_0 (\theta_1 + \gamma'_2 \theta_2)) \right)^+ - H_c \right)$ |
| $H_s$ | Stratiform heating | $\frac{\partial H_s}{\partial t} = \frac{1}{\tau_s} (\alpha_s H_d - H_s)$ |
| $D$ | Downdraft | $D = \frac{w_0}{Q} (1 + \mu (H_s - H_c))^+ (\theta_{eb} - \theta_{em})$ |
| $E_s$ | Surface flux | $\frac{1}{h} E_s = \frac{1}{\tau_e} (\theta_{eb}^s - \theta_{eb})$ |
Table 2: List of multicloud parameters and simulations. Note the values of $\tilde{Q}_j, j = 1, 2$ in parentheses correspond to a normalization with the $L_2$-norm (rms) of the basis functions $\phi_j, j = 1, 2$ that enter in the projections of the horizontal velocity field, used in the advection term of the moisture equation.

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<th>Parameter</th>
<th>MJO</th>
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<td>$\tilde{Q}_1$</td>
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<td>38.47 (6.15) K</td>
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