Simple multicloud models for the diurnal cycle of tropical precipitation. Part I: Formulation and the case of the tropical oceans

YEVGENIY FRENKEL

Courant Institute for Mathematical Sciences, New York University, New York, NY

BOUALEM KHOUIDER *

Department of Mathematics and Statistics, University of Victoria, Victoria, BC, Canada

ANDREW J. MAJDA

Center for Atmosphere-Ocean Sciences and Courant Institute, New York University, New York, NY.

*Corresponding author address: Boualem Khouider, Department of Mathematics and Statistics, University of Victoria, 3800 Finnerty Road, Victoria, British Columbia, Canada.

E-mail: khouider@uvic.ca
ABSTRACT

The variations of tropical precipitation due to the diurnal cycle of solar heating is examined here in the context of two simple models for tropical convection. The models utilize three cloud types, congestus, deep, and stratiform that are believed to characterize organized tropical convection and are based on the two first baroclinic modes of vertical structure plus a boundary layer mode. The two models differ mainly in the way they treat the boundary layer dynamics. The first one is purely thermodynamical and is reduced to a single equation for the equivalent potential temperature connecting the boundary layer to the upper troposphere through downdrafts and to the surface through evaporation while the second uses full bulk boundary layer (FBBL) dynamics with a careful separation between sensible and latent heat fluxes and parametrization of non-precipitating shallow cumulus. It turns out that in the case of the precipitation over the ocean where the Bowen ratio is small, both models yield a qualitatively similar solution, characterized by an overnight initiation and early morning peak in precipitation consistent with observations. The modelled diurnal cycle of precipitation over the ocean is divided into four cyclic phases. 1) A CAPE (re)generation phase characterized by the enhancement of the boundary layer $\theta_e$ and moisture fluxes during midday and early afternoon is followed by (2) a (re)moistening phase dominated by congestus heating during the late afternoon and moistening from downdrafts (due to detrainment of shallow cumulus, specifically in the FBBL model) and radiative cooling that lasts until midnight. 3) Deep convection is initiated around midnight when the middle troposphere is sufficiently moist and cool and (re)establishes the precipitation level near its radiative convective equilibrium (1 K/day) and then 4) peaks with sunrise at 6:00 am to yield a precipitation maximum of roughly 2 K/day at around 9:00 am that dries the troposphere
and consumes CAPE and closes the cycle.

1. Introduction

Seasonal and diurnal cycles of solar radiation have a major impact on the local and regional variability of Earth’s weather and climate. In particular, they induce significant variability, on those time scales, in tropical storms and the associated winds and precipitation. Early investigations of the diurnal variability of tropical precipitation over land and oceans date back, at least, as far as the early 20’s (Ray 1928) but it has not been one of the major research focus of scientists until recently, likely due to the advent of satellite measurements (especially GARP-GATE and TOGA-COARE) and computers (Krauss 1963; Wallace 1975; McGarry and Reed 1978; Reed and Jaffe 1981; Albright et al. 1981, 1985; Houze and Betts 1981; Sably et al. 1991; Hendon and Woodberry 1993; Sui et al. 1998). In fact, some of the most significant contributions to this field are triggered by the most recent satellite measurement from the Tropical Rainfall Measurement Mission (TRRM) (Yang and Slingo 2001; Takayabu 2002; Sorooshian et al. 2002; Zuidema et al. 2003; Smith and Rutan 2003; Nesbitt and Zipser 2003; Yang and Smith 2006; Kikuchi and Wang 2008). This important body of work led the community to important understanding of the diurnal variability of tropical precipitation over land and oceans. The most important of these findings are summarized and further clarified in Kikuchi and Wang (2008) thanks to their innovative use of empirical orthogonal functions (EOF’s) for the study of diurnal variations of precipitation in the tropics applied to two TRMM data sets supplemented by in situ measurements.
Kikuchi and Wang (2008) elucidated and confirmed the persistence of three main regimes in the diurnal variation of tropical precipitation. An oceanic regime, characterized by an early morning (6:00 am to 9:00 am local solar time: LST) peak of relatively weak amplitude is found over the deep waters of the three tropical oceans within the inter-tropical convergence zone (ITCZ), a continental regime with a late afternoon peak (3:00 pm to 6:00 pm LST) of large amplitude is found over central America and Africa, and a coastal regime is seen along the land-sea boundaries of the maritime continent of Indonesia and Sumatra, South India, West Africa, and Northeast of Brazil. The coastal regime is characterized by both a large amplitude peak and a propagation in both directions, perpendicular to the shoreline. Namely, there are two coastal regime categories. The land-side category associated with precipitation patterns that form in the afternoon to late evening (12:00 pm to 9:00 pm LST) and propagate landward and the sea-side category associated with precipitation patterns that form in the late evening to midnight (9:00 pm to 12:00 am LST) and propagate towards the deep sea.

The physical mechanisms that govern the three regimes of the diurnal cycle of tropical precipitation remain an open parameterization problem for global weather forecast and climate models, despite the noticeable progress of our understanding of some of these mechanisms, thanks to several theoretical and numerical simulation studies. While the land regime is associated to a direct thermodynamic response of the surface layer to solar radiation Kikuchi and Wang (2008) and the land-side coastal regime is thought to be due to landward propagation of meso-scale convective systems and tropical squall lines in the direction of the sea-breeze (Kousky 1980; Cohen et al. 1995; Garreaud and Wallace 1997; Kikuchi and Wang 2008), theories for both the oceanic and the sea-side coastal regimes remain very illusive.
(Yang and Smith 2006; Kikuchi and Wang 2008). The proposed mechanisms, for the later regimes, range, respectively, from shortwave to longwave radiation and from the concavity of the shoreline to the propagation of gravity waves. Furthermore, current global and regional numerical models of weather and climate have difficulty in reproducing the diurnal variability of tropical precipitation (Randall et al. 1991; Yang and Slingo 2001; Dai and Trenberth 2004; Tian et al. 2004) presumably, due to the misrepresentation of moist-tropical convection by the underlying cumulus parametrizations. Nevertheless, superparameterization and global cloud-resolving models seem to be able to capture the diurnal cycle of rainfall over tropical continents and oceans quite well Khairoutdinov et al. (2005); Sato et al. (2009).

In this series of two papers, we propose to use the multicloud model of (Khoudi and Majda 2006b, 2008, hereafter KM06, KM08) and its newer variant which is coupled to a full dynamical boundary layer (Waite and Khoudi 2009, hereafter WK09) to study the diurnal variability of tropical precipitation. In particular, the WK09 model is an important tool to contrast the role of various boundary layer processes such as sensible heat fluxes for the oceanic versus continental regimes. In this paper (Part I), we formulate the models, present the method of solution based on the applied mathematics technique of Floquet stability theory, combined with sophisticated numerical algorithms, for dynamical systems with periodic coefficients and discuss our results for the case of the tropical oceans. Part II addresses the case of continental precipitation where mild modifications of the multicloud model with a dynamical boundary layer of WK09, which allow for large temperature inversions at the top of the planetary boundary layer and emphasize the effect of sensible heat fluxes between the boundary layer and the free troposphere, are important new features in order to capture the continental regime of the diurnal cycle of precipitation over land.
The remaining of the paper is organized as follows. In section 2, we discuss briefly the two multicloud models, with and without boundary layer dynamics, named hereafter WK and KM models, respectively, forced from the surface by the diurnal cycle of solar heating. We use a Galilean transformation to write the corresponding systems in the local solar time (LST) coordinate system. For simplicity, we consider solutions for the diurnally varying climatology where the effects of waves and other spatial variations are ignored. The mathematical methodology used here to look for one-day periodic stable solutions, which combines sophisticated numerical methods of ordinary differential equations (ODEs) and the Floquet theory for nonlinear periodic solutions is presented in the Appendix. In section 3, we present the first set of results for the ocean case, characterized by no or very little contribution of sensible heat fluxes, i.e very small Bowen ratios (Hsu 1998; Sadhuram et al. 2001), where both the KM and the WK models show similar results, characterized by a morning precipitation peak consistent with observations. Sensitivity to model parameters and associated bifurcations from stable diurnal solutions to multi-day periodic or chaotic solutions are presented in section 4. Finally, a concluding summary and discussion is given in section 5.

2. Model formulation and Floquet stability theory

a. Governing equations

In this section we review briefly the dynamical and physical features of the multicloud model both with and without boundary layer dynamics used here to study the variability of
tropical precipitation due to the diurnal cycle of solar heating. Notice that only the most salient features of the multicloud modeling framework are presented here. A more complete discussion is found in the original papers (KM06, KM08, WK09).

The multicloud models assume three heating profiles that are associated with the main cloud types that are observed to characterize organized tropical convective systems (Johnson et al. 1999): Cumulus congestus clouds that heat the lower troposphere and cool the upper troposphere through radiation and detrainment, deep convective towers that heat the whole tropospheric depth, and the associated lagging-stratiform anvils that heat the upper troposphere and cool the lower troposphere due to evaporation of stratiform rain. Accordingly, the dynamical core of the multicloud model is based on the momentum and potential temperature equations for the first and second baroclinic modes of vertical structure that are directly forced by deep convection and both congestus and stratiform clouds, respectively (KM06, KM08, WK09). The multicloud models carry an equation for the vertically averaged moisture (water vapor mixing ratio), over the tropospheric depth, and bulk boundary layer dynamics—averaged over the atmospheric boundary layer (ABL).

In the case of the KM model, the ABL dynamics are reduced to a single equation for the boundary layer equivalent potential temperature forced by evaporation at the sea surface and downdrafts while the WK model assumes full dynamical equations for ABL horizontal velocity, potential temperature, and moisture and explicit representation of shallow cumulus and the associated entrainment and detrainment rates, following Stevens (2006). As a consequence of the ABL velocity equation, the free troposphere dynamics of the WK model has also a barotropic velocity (0th mode) with a non-zero horizontal divergence that matches the boundary layer convergence at the surface and decreases linearly to zero aloft (Biello
and Majda 2004, WK09).

For convenience, the governing equations, along the equatorial belt neglecting the effect of rotation and meridional extent, for the two models are reported in Table 1 while the associated constants and parameters are reported in Table 2. All equations are given in the non-dimensional form where the speed of first baroclinic Kelvin waves, $c_r \approx 50 \text{ ms}^{-1}$, is the velocity scale; the equatorial Rossby radius of deformation, $L_e \approx 1500 \text{ km}$, is the length scale; $T = L_e/c_r \approx 8.33 \text{ hr}$ is the time scale; and $\tilde{\alpha} = H_T N^2 \theta_0 / \pi g \approx 15 \text{ K}$ is the temperature unit scale.

The WK model expands the purely thermodynamic boundary layer in the KM model to include sensible heat fluxes, mixing due to shallow cumulus, and environmental downdrafts. Consequently, the boundary layer potential temperature $\theta_b$ and boundary layer moisture $q_b$ are handled separately. Though, for the case of the ocean considered here in Part I, the diurnal variations of precipitation in the WK model follow closely the results of the simpler KM model, separate treatment of sensible and latent heat is pivotal for the diurnal cycle over land which is discussed in the companion Part II paper. To facilitate the expression of parametrized effects of mixing associated with shallow convection and surfaces fluxes, we introduce the jump $\Delta_x \phi$, for a generic variable $\phi$, between the bulk boundary layer value $\phi_b$ and the value $\phi_x$ of $\phi$ at a given vertical layer $x$, where $x = s, t, m$ for the (sea or land) surface, the top of the ABL, and the middle of the troposphere, respectively. By convention, the jump is computed as the lower level value minus the higher level value so that $\Delta_s \phi = \phi_s - \phi_b$, $\Delta_t \phi = \phi_b - \phi_t$, $\Delta_m \phi = \phi_b - \phi_m$.

The surface (sea or land) values are prescribed while the values at the top of the ABL and in the mid-troposphere are determined according to the free tropospheric dynamics.
The effects of the diurnal solar cycle on ABL moisture and temperature appear in the model through $\Delta_s q = q_s - q_b$ and $\Delta_s \theta = \theta_s - \theta_b$. The effects of the diurnal cycle will be represented by considering perturbations to the surface values of potential temperature and boundary layer moisture.

$$q_s = \bar{q}_s + \hat{q}_s(t), \theta_s = \bar{\theta}_s + \hat{\theta}_s(t)$$  \hspace{1cm} (1)

where $t$ is time. In accordance with KM model, $\Delta_m \bar{\theta}_e = \bar{\theta}_{eb} - \bar{\theta}_{em}$ represents the discrepancy between boundary layer and mid-troposphere equivalent potential temperature. For WK model, we have the additional flux corresponding to $\Delta_m \bar{\theta} = \bar{\theta}_b - \bar{\theta}_m$, where $\gamma = -\Delta_m \bar{\theta}/\Delta_m \bar{\theta}_e$ – the ratio of the potential and equivalent potential temperature fluxes, which reveal to be crucial for the linear stability properties of the system. (The standard value is $\gamma = 0.5$).

Additionally, for the ocean case considered here, because of low Bowen ratio, we assume (in WK model) that $\Delta_s \bar{\theta} = 0$ at radiative convective equilibrium (RCE). In the same vein, we assume that the potential temperature inversion at the top of the boundary layer is zero: $\Delta_t \bar{\theta} = 0$ as our standard case. Sensitivity tests of the results to the effects of weak inversions and weak surface sensible heat fluxes are considered below.

Another critical feature of the multicloud models (KM06, KM08) is the use of a nonlinear switch $\Lambda$ which is a function of $\Delta_m \theta_e$, the difference between the the boundary layer potential temperature $\theta_{eb}$ and middle tropospheric potential temperatures $\theta_{em} = q + (2\sqrt{2}/\pi)(\theta_1 + \alpha_2 \theta_2)$, that allows natural transitions between congestus and deep convection. A dry middle troposphere impedes deep convection and promotes congestus clouds. A moist middle troposphere allows deep convection and subsequent stratiform clouds while impeding the formation of congestus clouds.
New to the present study, the WK model is altered to accommodate the redistribution of the entrainment and downdraft fluxes between the free tropospheric moisture and the potential temperature to allow penetration of sensible heat fluctuation through the top of the boundary layer interface, due to solar heating at the surface. In the original WK model, entrainment and downdraft fluxes entering both the ABL moisture and $\theta_b$ equations are balanced by similar fluxes in the free tropospheric moisture alone. Here, in Table 1, these fluxes are distributed between the moisture equation and the first baroclinic potential temperature by introducing the parameters $\alpha_E$ and $\beta_D$. Thus sensible heat arising from the ABL is allowed to warm the free troposphere due to cumulus entrainment. Note that $\alpha_E = \beta_D = 0$ corresponds to the original WK model. Here, we use $\alpha_E = \beta_D = 1/3$ as an appropriate choice based on the intuition that there is an asymmetry due sensible heat mixing by gravity waves. However, this effect turn out to be negligible for the ocean case, as long as the Bowen ratio is small, but it is crucial for the proper representation of the diurnal cycle of precipitation over land as it is demonstrated in the companion paper.

b. Diurnal cycle forcing and Galilean transformation

Latent heating is considered as one of the primary diurnal cycle forcing mechanisms over the tropical ocean. Bellenger et al. (2010) measured a rise in surface heat fluxes of around 50 W m$^{-2}$ and over, associated with diurnal variation of sea surface temperature. Here, the relationship between sea surface temperature (SST) and boundary layer saturation equivalent potential temperature perturbation $\hat{\theta}_{eb}$ are inferred from Clausius-Clapeyron equation. Recent in situ measurements show that diurnal variations of the skin sea surface temperature
over the tropical oceans range roughly from 2° C to 3° C, depending on various factors (Zeng et al. 2010; Bellenger et al. 2010). Accordingly, we consider a maximum amplitude of the $\theta_e$ perturbation based on a maximum perturbation of up to 3° C, from an SST average varying between $T = 25° C$ and $T = 28° C$:

$$\hat{\theta}_{eb,max}^\star = 3° C + \frac{L_v}{C_p}[q^\star(T + 3° C) - q^\star(T)] \quad (2)$$

Values of latent heat and air pressure at 25° C yield maximum perturbations $\hat{\theta}_{eb,max}^\star = 11.76° C$ and $\hat{\theta}_{eb,max}^\star = 13.21° C$, for $T = 25° C$ and $T = 28° C$, respectively. Here we use $\hat{\theta}_{eb,max}^\star = 10° C$ as a conservative value for the ocean. To accommodate the asymmetry between the day time solar heating and night time radiative cooling, we assume diurnal variation of SST in the form of a half-sine profile so that

$$\hat{\theta}_{eb}^\star(x,t) = \hat{\theta}_{eb,max}^\star \begin{cases} \frac{\pi}{\pi - 1} \left( \sin(x + ct) - \frac{1}{\pi} \right) & \text{if } x + ct \mod(2\pi) < \pi \\ -\frac{1}{\pi - 1} & \text{if } x + ct \mod(2\pi) > \pi \end{cases}$$

(3)

see also Fig. 2.

The nature of the diurnal cycle forcing suggests an appropriate coordinate reference frame that circles the globe in one day following the peak of the solar energy travelling at an average speed of $c = 463 m/s$. We consider a Galilean transformation for the equations of motion on the form $x_g = x + ct$, $\tau = t$, where $x_g$ is in the coordinate system that moves with the local solar time (LST). Notice that accordingly variations is $x_g$ can be interpreted as relating to different locations on the equator during the same time of the day or relating to a fixed location at different times of the day, i.e LST. The later convention is adopted throughout this paper. Using the chain rule, the time and space derivatives are transformed,
respectively, into

\[ \frac{\partial}{\partial t} = c \frac{\partial}{\partial x_g} + \frac{\partial}{\partial \tau} \] and \[ \frac{\partial}{\partial x} = \frac{\partial}{\partial x_g} \]

in the new variables \(x_g\) and \(\tau\). To focus on the diurnal variation of the background climate, without consideration of wave activity and other flow complexities, we seek solutions \(U(x_g, \tau)\) that do not depend on the new temporal parameter \(\tau\). The resulting systems of equations that are explicitly solved here for both the KM and WK models are given next.

The KM model for background variations due to diurnal cycle of SST is given by

\[
\begin{align*}
    u'_1 &= \frac{-c}{c^2 - 1} (C_d u_0 + \frac{1}{\tau_R}) u_1 + \frac{1}{c^2 - 1} (H_d + \xi_s H_s + \xi_c H_c - \frac{\theta_1}{\tau_D} - Q^0_{R,1}) \\
    u'_2 &= \frac{-4c}{4c^2 - 1} (C_d u_0 + \frac{1}{\tau_R}) u_2 + \frac{4}{4c^2 - 1} (-H_s + H_c - \frac{\theta_2}{\tau_D} - Q^0_{R,2}) \\
    \theta'_1 &= \frac{c}{c^2 - 1} (H_d + \xi_s H_s + \xi_c H_c - \frac{\theta_1}{\tau_D} - Q^0_{R,1}) + \frac{-1}{c^2 - 1} (C_d u_0 + \frac{1}{\tau_R}) u_1 \\
    \theta'_2 &= \frac{4c}{4c^2 - 1} (-H_s + H_c - \frac{\theta_2}{\tau_D} - Q^0_{R,2}) + \frac{-1}{4c^2 - 1} (C_d u_0 + \frac{1}{\tau_R}) u_2 \\
    q' &= \frac{1}{c + u_1 + \alpha u_2} [-q(u'_1 + \tilde{\alpha} u'_2) - \tilde{Q}(u'_1 + \tilde{\lambda} u'_2) - P + \frac{D}{H_T}] \\
    H'_s &= \frac{1}{c \tau_s} (\alpha_s H_d - H_s) \\
    H'_c &= \frac{1}{c \tau_c} (\alpha_c (\Lambda Q_c^+) - H_c) \\
    \theta'_{eb} &= \frac{1}{c} \left( \tau_e^{-1} (\tilde{\theta}_{eb} - \bar{\theta}_{eb} + \tilde{\theta}_{eb}(x_g) - \theta_{eb}) - \frac{1}{h_b} D \right)
\end{align*}
\]
while the WK model is given by

\[ c\bar{u}' = -\bar{u}u' - p_0' - \bar{u}'(\bar{u} - u_b) + \frac{E_u}{H_T} \Delta_t u \]

\[ cu_j' = -\bar{u}u_j' - u_j\bar{u}' - \theta_j' - u_j/\tau_R + \frac{\sqrt{2}}{T_T} \delta_b \Delta_t u \]

\[ c\theta_1' = -\bar{u}\theta_1' + u_1' - \sqrt{2}\bar{u}' + \frac{\pi}{2\sqrt{2}} (P + \alpha_E \frac{E}{H_T} \Delta_t \theta + \beta_D \frac{M_d}{H_t} + \bar{u}') \Delta_m \theta - Q_{R1} - \theta_1/\tau_D \]

\[ c\theta_2' = -\bar{u}\theta_2' + \frac{1}{4} u_2' - \frac{\sqrt{2}}{4} \bar{u}' - H_s + H_c - Q_{R2} - \theta_2/\tau_D \]

\[ cq' = -\bar{u}q - [(u_1 + \delta u_2)q + (u_1 + \tilde{\lambda} u_2)\tilde{Q} - \bar{u}\tilde{Q}_0]' - P \]

\[ + \frac{E}{H_T} \Delta_t ((1 - \alpha_E) \theta + q_b) + \frac{M_d}{H_t} + \bar{u}')) \Delta_m ((1 - \beta_D) \theta + q_b) \]

\[ cu_1' = -u_1 u_1' - p_1' - \frac{E_u}{h_b} \Delta_t u - \frac{C_d U}{h_b} u_b. \]

\[ p_0 = p_b + \sqrt{2}(\theta_1 + \theta_2) + \frac{\pi}{2} \delta_b \theta_b \]

\[ (1 + \delta_b)p_b' = 2\delta_b \bar{u}'(\bar{u} - u_b) - \sqrt{2}(\theta_1' + \theta_2') - \frac{\pi}{2} \delta_b \theta_b' - \delta_c \frac{C_d u_0}{h_b} u_b \]

Here the prime denotes the derivative with respect to the variable \( x_g \), which is the only independent variable in these systems while the dependent variables are as in the previous subsection and Table 1. Notice that the diurnal forcing enters the KM equations (4) via the prescribed periodic perturbation \( \hat{\theta}_{eb}(x_g) \) appearing of the right hand side of the \( \theta_{eb} \) equation while the WK equations (5) are forced from both the \( \theta_b \) and the \( q_b \) equations through the specific definitions of the jumps \( \Delta_s \theta \) and \( \Delta_s q \) that depend on the diurnal perturbations of the sea surface temperature \( \hat{\theta}_s(x_g) \) and saturation moisture \( \hat{q}_s(x_g) \) computed according to Clausius-Clapeyron equation and the imposed maximum SST perturbation of 3° C with the imposed half-sine profile in (3).
Because of the quasi-linearity of the KM model (the nonlinear terms are independent of
the solution derivatives), the system in (4) is written on the form $U' = F(U, x_g)$. This is not
the case for the WK model.

Notice that the relatively large value of the propagation speed $c$ suggests the possibility
for easy asymptotic analysis. For example, it easy to see that the dynamical core of the KM
model decouples almost completely leaving the potential temperature equations independent
of the velocity field. In fact, in the velocity equations, the contributions from field fields is
scaled by $1/c^2$ and thus the velocity field is only weakly forced by the diurnal cycle of
solar heating and is not expected to grow in the absence of waves and other spatial non-
homogeneities.

The mathematical tools used to search for stable periodic solutions, consisting of sophis-
ticated numerical algorithms and the Floquet stability theory, for the two ODE systems
(4) and (5) are reported in the Appendix. The Floquet multipliers, which are basically the
eigenvalues of the fundamental matrix of first oder variation of periodic solution, determine
the stability of the system when the Floquet theory is applicable. If all Floquet multipliers
are inside the unit (complex) circle the associated periodic solution is asymptotically sta-
bile. It is unstable if at least one eigenvalue is outside the unit circle (see the Appendix for
details).
3. Model results and comparison to observations: The ocean case

a. The typical stable one-day periodic solution

As mentioned above our search for stable periodic solutions is guided by the linear stability analysis of the steady homogeneous RCE state. When the surface flux is constant ($\dot{\theta}_{eb}^* \equiv 0$) the multicloud equations (4) and/or (5) admit time-independent homogeneous steady state solutions (KM06, KM08). Linear stability analysis of such equilibrium points is standard in the theory of autonomous ODE systems, $u' = f(u)$, which consists of studying the eigenvalues of the constant coefficient matrix formed by the Jacobian determinant of the vector-field $\nabla_u f(u)$. The stability diagram associated with such RCE state for the KM model is plotted in Fig. 1, for two different values of the background state, $\bar{\theta}_{eb} - \bar{\theta}_{em} = 12$ K and $\bar{\theta}_{eb} - \bar{\theta}_{em} = 16$ corresponding to a moist and a dry mid-troposphere, as a function of two key parameters: the strength of the surface flux $\theta_{eb}^* - \bar{\theta}_{eb}$ at RCE and the fraction stratiform rain that reaches the surface, $f_s$ (KM08), on the two top panels, and the congestus adjustment coefficient $\alpha_c$, on the two bottom panels. Note that $\theta_{eb}^* - \bar{\theta}_{eb}$ is varied from zero to $20^\circ$C to cover the region in parameter space in which this flux would oscillate following a diurnal cycle with an associated perturbation of $10^\circ$ C around a mean of the same magnitude.

Interestingly, on the top panels in Fig. 1, for $\bar{\theta}_{eb} - \bar{\theta}_{em} = 12$ K, the homogeneous RCE is stable for all values of $\theta_{eb}^* - \bar{\theta}_{eb}$, for all the physically reasonable values $f_s < 1$, while the dry RCE case $\bar{\theta}_{eb} - \bar{\theta}_{em} = 16$ K is stable for roughly all $\theta_{eb}^* - \bar{\theta}_{eb} < 15$ K and unstable when $\theta_{eb}^* - \bar{\theta}_{eb} \geq 15$ K. This suggests that periodic solutions induced by diurnal variations of the
surface fluxes roughly spanning the horizontal axis on the top panels of Fig. 1, corresponding to a moist state around \( \bar{\theta}_{eb} - \bar{\theta}_{em} = 12 \) K will be stable while those corresponding to the dry state \( \bar{\theta}_{eb} - \bar{\theta}_{em} = 16 \) K may or may not be stable given that the diurnal cycle is constantly forcing the solution back and forth from the stable and the unstable regions, obviously depending on the residence time of the solution in each region weighted by the damping and growth rates, respectively. Interestingly, in the dry case, the instability region seems to decrease monotonically with the parameter \( f_s \). The stratiform rain seems to have a stabilizing effect in the model dynamics. The same remarks apply for the case when \( \alpha_c \) varies instead of \( f_s \) on the two middle panels.

These intuitive arguments are confirmed on the bottom panels of Figure 1 where a periodic solution corresponding to the surface diurnal forcing in (3) for an arbitrary value of the parameter \( 0.1 \leq \alpha_c \leq 0.4 \) is found and its largest Floquet multiplier is computed according to the methodology described in the Appendix. From the bottom panels in Figure 1, we see that consistent with the intuition motivated above, the periodic solution of the KM system (4) is Floquet stable for roughly \( \alpha_c \leq 0.33 \) (\( \alpha_c \leq 0.23 \)) and unstable otherwise when \( \bar{\theta}_{em} - \bar{\theta}_{eb} = 14 \) K (\( \bar{\theta}_{em} - \bar{\theta}_{eb} = 16 \) K).

A typical stable periodic solution associated with the diurnal cycle forcing, for the standard parameter values reported in Table 2 is plotted in Figs. 2 and 3 for the KM and WK models, respectively, for the case of a moist RCE background state corresponding to \( \bar{\theta}_{eb} - \bar{\theta}_{em} = 12 \) K and \( f_s = 0.4 \). The perturbation of saturation equivalent potential temperature used for the KM model is given by the dashed line on the bottom right corner of Fig. 2. Note that the zero surface flux of sensible heat assumed for the WK model is consistent with the weak Bowen ratios found over the tropical oceans (Hsu 1998; Sadhuram et al. 2001).
The boundary layer potential temperature is still forced externally through the entrainment and downdraft fluxes. However, no significant changes in results (not reported here) were observed when surface sensible heat flux perturbations of up to $\hat{\theta}_{s,\text{max}} = 1$ K were allowed. Also a temperature inversion of up to $\Delta t\bar{\theta} = \Delta t\bar{\theta} = -2$ K at the top of the boundary layer yields qualitatively similar results.

From Figs. 2 and 3, we first note that consistent with the asymptotic predictions conjectured above, the velocity components of the periodic solutions shown on the top left panels of Figs. 2 and 3, are very weak and are more so for the KM model because of the lack of boundary layer dynamics. Therefore we confirm the tautology that without wave activity and other spacial variations, the interaction of the diurnal cycle with convection alone cannot produce the strong winds observed in nature. However, the most important feature that is common to the two models is the persistence of a morning precipitation peak on the bottom left panels of Figs. 2 and 3, consistent with the deep convective heating, $H_d$. Note that the stratiform and congestus rain fractions are set to $f_c = 0.1$ and $f_s = 0.4$, respectively, to allow a substantial contribution to the total precipitation from these two cloud types.

The variability exhibited by Figs 2 and 3 is qualitatively consistent with observations in the tropical oceans (Yang and Slingo 2001, Kikuchi and Wang 2008), where diurnal precipitation, averaged on the climatological scale, starts around midnight, peaks in the morning between 6 am and 9 am LST and reaches its minimum in the afternoon and late evening. The modelled precipitation seems to follow the same pattern with the precipitation maximum around 10:30 am for the KM model and around 9:30 am to 10:00am for the WK model. Interestingly, the two precipitation patterns are self-similar but the WK model seems to lead by about an hour or so, except for the sudden acceleration of precipitation
that coincides with sunrise at 6:00 am which happens at the same time for both models.

b. The physical mechanism of diurnal cycle of precipitation over the ocean

The underlying physical mechanism and dynamical behavior associated with the solutions in Figs. 2 and 3 is explained here in terms of the interactions of the three cloud types with the periodically forced boundary layer dynamics. It is evident from the plots in Figs. 2 and 3 that the sudden acceleration of the morning deep convection is fueled by the increase of moisture in the ABL. The morning sunrise triggers an abrupt increase of the saturation equivalent boundary layer temperature which drives the increase in equivalent potential temperature of the boundary layer. This results in an increase in convectively available potential energy (CAPE), i.e., the maximum energy available for deep and congestus convection ($Q_d$ and $Q_c$) in the multicloud model. At night, in the absence of solar heating, the troposphere reaches its radiative convective equilibrium soon after the atmosphere is cool and moist enough, due to longwave radiation and moistening by boundary layer fluxes. Note the increase of free troposphere moisture and the significant decrease of potential temperature $\theta_1$ during the evening between roughly 4:00 pm and midnight. While the decrease in temperature is clearly due to the imposed radiative cooling of 1K/day (resulting roughly in 0.3 K cooling during these 8 hours), the increase in moisture is more subtle. Moisture is pumped to the boundary layer during midday and afternoon due to evaporation (resulting in the rise of the $\theta_{eb}$ and $q_b$ curves on the bottom right panels of Figs. 2 and 3, respectively) and progressively transferred to the free troposphere during the late afternoon and evening due to downdrafts and entrainment fluxes (that model mixing due to shallow convection activity). This is
obviously reflected by the increase of the $q$ curves on the middle left plots of Figs. 2 and 3 during this period. The RCE state is characterized by moderate precipitation rate of roughly 1 K/day (that nearly balances the imposed radiative cooling) that plateaus between roughly midnight and 6:00 am.

Given the preconditioned, high relative humidity, free troposphere at night (necessary for deep convection), the sudden rise in CAPE forces deep convection to rise above the RCE level as soon as $\theta_{eb}$ starts increasing and peaks shortly after 9 AM when $\theta_{eb}$ becomes positive. Deep convection quickly dries out and warms the atmosphere. The warm and dry atmosphere (with low relative humidity) is not efficient at sustaining deep convection and thereby precipitation declines and marks the morning maximum. Additionally, the stratiform clouds lagging deep convection cool the lower troposphere. (Recall that $Q_c$ is highly sensitive to the second baroclinic potential temperature through higher value of $\gamma_2'$ introduced in KM08). The stratiform cooling combined with the rise of $\theta_{eb}$ induce a (re)generation of CAPE, while the mid-troposphere is still dry, promotes an afternoon congestus peak. The afternoon congestus likely helps moisten the middle atmosphere through low level moisture convergence but it doesn’t seem to be the main mechanism here, in the absence of wave activity, as moisture continues to rise during the evening before deep convection (at RCE level) occurs overnight.

The one hour leading of the precipitation pattern in Fig. 3 compared to Fig. 2 is associated to an enhancement of the moisture fluxes at the top of the boundary layer interface in the WK model due to the explicit representation of shallow cumulus entrainment rate (the term containing the factor $E$ in the $q$-equation in (5)), allowing an earlier preconditioning time.

According to the present model results, the diurnal cycle of precipitation over the tropical
Ocean is divided into a cycle of four phases: (1) A CAPE (re)generation phase characterized by the rise of $\theta_{eb}$ during midday and early afternoon is immediately followed by (2) a moistening and cooling phase starting by congestus heating in late afternoon and continues to moisten until around midnight due to the detrainment of shallow cumulus clouds. (3) Once the atmosphere is sufficiently moist, deep convection is initiated around midnight and quickly establishes a near RCE state. This episode can be termed as the initiation and RCE phase. Finally, (4) the precipitation peak phase starts with sunrise at 6:00 am to maximize at around 9:00 am, which dries the troposphere and consumes CAPE at the same time by heating the upper troposphere and cooling and drying the boundary layer through downdrafts.

c. Case of a dry RCE

In Figs. 4 and 5, we repeat the experiments in Figs 2 and 3 for the case of a dry atmospheric background corresponding to $\bar{\theta}_{eb} - \bar{\theta}_{em} = 16$ K. We note that although, the stability diagram in Fig. 1 suggest that the perturbed RCE visits the unstable region a considerable amount of time, at all times of the day when $\theta^*_{eb} - \theta_{eb} \geq 15$ K, the periodic solutions reported in Figs. 4 and 5 are asymptotically stable by Floquet stability analysis.

The two solutions in Figs. 4 and 5 look astonishingly similar to their analogs in Figs. 2 and 3, except for the stronger afternoon congestus peak, which is more so for the WK model in Figure 5, consistent with the drier atmosphere. The stronger congestus peak contributes a fair amount to the total precipitation which thus appears to have a secondary afternoon peak, especially for the case of the KM model in Fig. 4. Consistently, this secondary peak
extends the drying period of the troposphere and therefore delays the re-establishment of the nighttime RCE-level precipitation. Double precipitation peaks or semidiurnal variability of precipitation are common in nature especially over land and coastal regions (Kikuchi and Wang, 2008; due to the variability carried by EOF3 and EOF4 in their analysis), however we don’t claim that our model results here are necessarily applicable to those particular cases.

Two interesting questions are whether the secondary-congestus peak will ultimately become dominant, especially for the WK model, as the atmosphere becomes considerably drier and when the “bifurcation” to the double peaks appears as $\bar{\theta}_{eb} - \bar{\theta}_{em}$ is increased between the 10 K to 20 K thresholds. In Figure 6, we plot the total precipitation contours in the time-$\bar{\theta}_{eb} - \bar{\theta}_{em}$ axises for the KM and WK model for the same parameter as in Figs. 2 to 5 above. It appears that for both models, for $\bar{\theta}_{eb} - \bar{\theta}_{em} \leq 16$ K or so, the timing of the precipitation peak is insensitive to the tropospheric background dryness and the that the transition to a regime with a secondary congestus peak appears simultaneously when $\bar{\theta}_{eb} - \bar{\theta}_{em}$ crosses the critical value of 16 K.

Except for the slightly earlier morning peak, the WK model provides similar results as the KM model despite the extra-level of complexity due to the explicit representation of shallow cumulus convection and the boundary layer dynamics. Therefore, in the remainder of this paper, we use only the KM model for the sake of simplicity. The WK model will be exclusively used for the case of the diurnal cycle over land in the companion paper.
4. Sensitivity to parameters

a. Convective heating parameters

While the role of congestus and stratiform clouds is essential in the multicloud model, perhaps the most important parameters to vary in the multicloud model are those involved in the deep convection closure, namely the coefficients $a_1$, $a_2$ and $a_0$ of $\theta_{eb}, q$, and $\theta_1 + \gamma_2 \theta_2$, respectively (see $Q_d$-equation in Table 1). Deep convection increases with increasing $\theta_{eb}$ and $q$, and decreasing $\theta_1 + \gamma_2 \theta_2$, that is, a moister and warmer boundary layer, as in CAPE parameterizations, a moister troposphere, as in Betts-Miller-type schemes, and a cooler lower middle troposphere, which favors saturation at weaker mixing ratios, all lead to a potential increase of the potential for deep convection. The first two parameters satisfy $(a_1 + a_2 = 1)$ and serve as a switch between CAPE and Betts-Miller parameterization, while the last parameter $a_0$ parametrizes the convective response to fluctuations in the dry buoyancy (KM06).

Fig. 7 illustrates the diurnal precipitation for parameter regimes with different contributions of boundary layer $\theta_e$ to deep convection ($a_1 = 0, 0.1, 0.45, 0.9, 1$). The most peculiar result is seen in the extreme case when $a_1 = 0$ (corresponding to a pure Betts-Miller-type parametrization, where deep convection is governed solely by mid-troposphere temperature and moisture anomalies). It is characterized by a weak afternoon precipitation peak, which is dominated by the contribution of congestus clouds, that does not exceed 1.5 K/day, except for the extremely dry RCEs at $\bar{\theta}_{eb} - \bar{\theta}_{em} \geq 16$ K. This congestus dominated afternoon precipitation peak should not be associated with the diurnal cycle over land which is characterized by large deep convection. Full boundary layer dynamics and associated sensible heat fluxes
are required in order to capture the late afternoon precipitation peak associated with the land regime as shown in Part 2.

A transition to a morning precipitation peak is seen, at sufficiently moist RCEs, for values of $a_1$ as small as $a_1 = 0.1$. This is consistent with the earlier interpretation in section 3 where boundary layer $\theta_e$ induces an increase in deep convection with the morning sunrise (as illustrated in Fig. 6) in addition to the overnight initiated deep convection, marking the early morning precipitation peak. Even small values of $a_1$ produce a physically compelling sharp increase in convection at 6:00 am. However, for small values of $a_1$, the structure of the peak itself is very flat. Moreover, the afternoon congestus precipitation peak is more pronounced for moist cases compared to the standard parameter regime shown in Fig. 2. This is mainly due to the fact that the maximum available energy for congestus heating (low level CAPE) depends strongly on ABL equivalent potential temperature. As the ABL equivalent potential temperature rises during the day, the model produces abundant congestus clouds, as the mid-troposphere remains relatively dry.

For parameter regimes with large $a_1$ (corresponding to CAPE type parametrization), the deep convection potential is mainly controlled by $\theta_{eb}$ and free tropospheric temperature. For $a_1 = 0.9$ and $a_2 = 0.1$, the deep convective precipitation occurs only during the day. This is due to the weak or no dependence of deep convection potential on moisture; as the strong precipitation episode dries out the boundary layer and consumes CAPE, there is no other mechanism to restore deep convective precipitation to the RCE value prior to sunrise and this results in a delayed morning precipitation peak. As the mid-troposphere gets drier (larger $\bar{\theta}_{eb} - \bar{\theta}_{em}$), the maximum in the precipitation is shifted towards noon as seen on the bottom of Fig. 7.
b. Sensitivity to stratiform rain fraction

This section considers the effect of the stratiform rain fraction on the diurnal cycle of precipitation for the KM model. Recall that in KM08 the rain fractions due stratiform and congestus cloud types are given, as averages relative to variations in the moisture switch function $\Lambda$, by

\[ f_s = \xi_s \alpha_s (1 + \xi_s \alpha_s + \xi_c \alpha_c)^{-1} \quad \text{and} \quad f_c = \xi_c \alpha_c (1 + \xi_s \alpha_s + \xi_c \alpha_c)^{-1}. \]

The standard values used in this paper $f_s = 0.4$, $f_c = 0.1$ are chosen in accordance with the observational data according to which congestus clouds account for 10% of total precipitation, while stratiform and deep convective precipitation contribute about 40% and 50%, respectively (Schumacher and Houze 2003). However, variations in stratiform rain fraction ranging from 25% to 85% are observed in TRMM data. In general, land regions tend to have lower stratiform rain fractions and higher deep convective activity. Stratiform rain fractions are lowest (20%-30%) over central Africa, the Bay of Bengal and eastern Brazil but the highest stratiform rain fractions (55%-60%) can be found over the intertropical convergence zone (ITCZ) over the eastern-central Pacific. This stratiform rain contrast, between the two regions of the globe, is believed to have a profound impact on the Walker circulation.

While the standard value of $f_s = 0.4$ used throughout the paper is a good approximation on average, large variations in stratiform rain fractions ranging from $f_s = 0.1$ to $f_s = 0.7$ are considered here to mimic the observations. In Fig. 8, we plot the diurnal variation of the deep convective precipitation only, without the contribution from stratiform and congestus clouds, for the three stratiform fractions, $f_s = 0.1$ (top), $f_s = 0.4$ (middle), and $f_s = 0.7$ (bottom), as a function of $\bar{\theta}_{eb} - \bar{\theta}_{em}$. We note that the deep convective precipitation peak shifts toward 10:00 am and is higher for the lower stratiform rain fractions, especially for
small values of $\bar{\theta}_{eb} - \bar{\theta}_{em}$ (moist RCE states). This is consistent with observations of Yang and Slingo (2001) who associate the ITCZ with the earlier precipitation peak while Schumacher and Houze (2003) make a connection between the ITCZ and a high amount of stratiform clouds. Interestingly, the case with $f_s = 0.7$ exhibits a double peak: one right after midnight (around 2:00 am) and one early morning around 8:00 am. Nonetheless, the total precipitation $(H_d + \xi_s H_s + \xi_c H_c)$ looks unchanged, in both profile and magnitude, with all three cases look almost identical to the middle panel in Figure 6. The decrease of the deep convective amplitude associated with the large stratiform fractions seem to be compensated by the lagging stratiform precipitation to provide a consistent total precipitation pattern which is insensitive to the stratiform fraction. This provide further evidence for the robustness of the physical mechanism, in the form of a four phase cycle, that governs the diurnal cycle of precipitation, in the model as described in the previous section and summarized in the conclusion section below.

c. Floquet unstable solution and bifurcation to multiday periodic and chaotic solutions

Even though the most physically relevant, standard parameters used in this paper produce nonlinear one-day periodic solutions that are Floquet stable, we can find Floquet unstable solutions, when those parameters are pushed toward some extreme values. Interestingly, these Floquet unstable solutions often evolve into multi-day periodic solutions as well as chaotic states, when integrated forward in time, somewhat similar to the behavior observed in (Khouider and Majda 2006a) for homogeneous background states without diurnal cycle forcing. The study of such solutions may provide some insight into the behavior of the model.
in the presence of convectively-coupled waves.

One important parameter that exhibits such behavior is the congestus adjustment coefficient $\alpha_c$. (See Table 1). Linear stability analysis of the homogeneous basic RCE state, shows that large $\alpha_c$ values (roughly $\alpha_c \geq 0.3$) lead to RCE’s with large regions of linear instability in the $(\theta_{eb}^* - \bar{\theta}_{eb})-(\bar{\theta}_{eb} - \bar{\theta}_{em})$ plane (as shown on the bottom of Figure 1). Floquet instability emerges when the diurnal cycle forcing drives the system, most of the time, into the linearly unstable RCE regions in terms of perturbations in $\theta_{eb}^* - \bar{\theta}_{eb}$. To illustrate this behaviour, Fig. 9 shows a chaotic solution that evolved from a Floquet unstable one-day periodic solution corresponding to the value $\alpha_c = 0.4$ and $\bar{\theta}_{eb} - \bar{\theta}_{em} = 14$ K. The dashed lines, in the background, represent the corresponding unstable one-periodic solution. Interestingly, the loss of periodicity is mostly exhibited in the congestus heating plot. The chaotic solution seems to evolve from days with strong congestus activity in the afternoon to days with zero congestus to days with moderate congestus, etc. Consistent with the plots on bottom of Fig. 1, the bifurcation to the chaotic behavior in Fig. 9 evolves gradually from the one-day periodic solution to two-day and subsequently four-day periodic solutions, as the parameter $\alpha_c$ is increased from roughly $\alpha_c = 0.33$ to higher values. This is consistent with the period doubling bifurcation associated with a Floquet multiplier crossing the unit circle at minus one (Dijkstra 2005). Interestingly, those multi-day solutions look very similar to the chaotic solution depicted in Fig. 9 with the most significant extra-diurnal variability exhibited in the congestus heating and have otherwise a somewhat consistent and persistent diurnal cycle of precipitation characterized by a morning peak, etc.
5. Concluding summary and discussion

The multicloud model of Khouider and Majda (2006, 2007, 2008a, 2008b, etc.) and its extended version that includes more detailed boundary layer dynamics (Waite and Khouider 2009) are used here to study the diurnal cycle of precipitation over the tropical ocean. The main multicloud model consists of the two vertical baroclinic modes of vertical structure forced by heating profiles consistent with the three cloud types that characterize organized tropical convection: cumulus congestus, deep convection, and trailing stratiform cloud decks, coupled to a thin boundary layer that responds to sea-surface heating and downdrafts. While the boundary layer dynamics in the original multicloud model are reduced to a single equation for the equivalent potential temperature (KM06, KM08), the WK09 version incorporates the bulk-boundary layer equations of Stevens (2006) and thus facilitates the treatment of sensible versus latent heat fluxes separately and entrainment and detrainment fluxes associated with shallow cumulus. However, since the Bowen ratio over the tropical ocean remains small, the two models yield qualitatively the same results, characterized with overnight precipitation followed by a weak morning peak, consistent with observations Kikuchi and Wang (2008).

The equations of motion along the equator—without rotation and no meridional dependence, forced by a diurnal cycle forcing, from the sea surface, are written in a moving frame circling the globe at a constant speed of one rotation per day following the local solar time (LST) to facilitate the analysis. Time independent nonlinear solutions depicting the diurnal variations of the climatology where the effects of waves and other non-homogeneities are filtered out are found in the form of stable periodic solutions using Floquet theory (Hochstadt 1975). The search for such stable periodic solution was guided by linear stability analysis of
the homogeneous basic state (no diurnal cycle forcing).

The present study highlights the importance of capturing interactions between the three different cloud types and in particular the effect of moisture preconditioning for the diurnal cycle of precipitation over the ocean. Even though moisture is abundant in the ABL during the day, deep convection requires high relative humidity of the free troposphere. In the absence of free tropospheric moisture, the model inhibits deep convection and favors congestus clouds when CAPE is positive. Detrainment of shallow cumulus and cumulus congestus is the main mechanism for transporting the ABL moisture into the free atmosphere and thereby helps create a favorable environment for deep convection, together with radiative cooling.

The main result of this paper, consisting of a typical stable one-day periodic solution that exhibits a realistic variability of the diurnal cycle of precipitation over the tropical ocean, is depicted in Figs. 2 and 3 for the KM and the WK models, respectively. According to these figures (that are qualitatively similar), the diurnal cycle of convection over the tropical ocean can be broken into four cyclic-main phases, which may overlap in duration: 1) CAPE (re)generation phase: The sun heated sea surface at midday induces an increase of both CAPE and boundary layer flux of moisture (right-bottom panel of Fig. 2). 2) Congestus and low-level moistening phase: The extremely dry mid-troposphere, following the morning precipitation peak, favors congestus clouds in the late afternoon (dashed line on the right-middle panel of Fig. 2) at the expense of deep convection and the re-moistening phase starts at around 3:00 pm, before the peak in congestus heating, and continues until midnight thanks to downdrafts due to shallow cumulus (explicitly represented in the WK09 model). 3) RCE level precipitation phase: After the preconditioning phase, deep convection
initiates at around midnight, when the mid-troposphere is sufficiently moist, and it quickly establishes a near RCE level. 4) Heavy deep convection precipitation phase: Deep convection peaks up with sun rise, at 6:00 am in the morning and reaches it maximum peak at around 9:00 am and quickly dries the middle troposphere and at the same time consumes CAPE to allow stage one to repeat the cycle.

These four phase characterization is roughly consistent with observations. In addition to the overall picture of capturing the typical morning precipitation peak reported in many observational studies (Kikuchi and Wang 2008), the present model captures important salient features of the diurnal cycle of tropical convection, over the ocean, including the phasing between variables consistent with the four phases reported above. The pronounced congestus heating in the afternoon, in phase 2, is consistent with shallow cumulus and cumulus congestus clouds peaking during the afternoon over the Indian Ocean, according to recent results by Bellenger et al. (2010), who also demonstrated persistent minimum precipitation afterwards i.e. during the evening as well as a rise of CAPE around 3:00 pm LST during the days with a warm boundary layer. The two last features are consistent with the zero precipitation recorded by the model after roughly 9:00 pm and the rise of $\theta_{eb}$ (which can be used as a proxy for CAPE anomalies, provided the upper troposphere is not too warm) during the afternoon as reported on Fig. 3 for example.

However, the lack of an active radiation scheme in the model impedes the inclusion of the diurnal variation in radiative heating, due to the absorption of sunlight by water vapor and clouds, which constitutes another major diurnal cycle forcing of the tropical atmosphere. Such radiation scheme will be developed and incorporated into the multicloud model by the authors in the near future, to assess the effect of diurnal radiative forcing on the diurnal
cycle of tropical precipitation in the model.

A case of relatively dry mid-troposphere RCE state with $\bar{\theta}_{eb} - \bar{\theta}_{em} = 16$ K is considered in Fig. 4 and 5 (see also Fig. 6), for the KM and the WK models, respectively. Interestingly, the solutions look very similar to their analogues in Figs. 2 and 3, respectively, except for the expected stronger congestus heating in the afternoon. This experiment suggests that regions of the tropical ocean that are characterized by a relatively dry mid-troposphere may have a secondary afternoon precipitation peak (as in Fig. 5) due essentially to congestus cloud decks which should not be confused with the dominating and much stronger afternoon precipitation peak over land which is driven by deep convection as it is captured by the WK model (see Part 2).

Sensitivity of the results to severe changes in some key parameters, considered in section 4, demonstrate that the physical mechanisms that govern the model’s diurnal cycle of precipitation are very robust. When for instance the stratiform rain fraction, $f_s$, takes a large value deep convection precipitation becomes very weak and exhibit a double peak, one before sunrise around 3:00 am and one after sunrise around 8:00 am. But the total precipitation looks astonishingly similar to the ones associated with smaller $f_s$ values. This is an important test for the model since regions that are dominated by stratiform rain are abundant in the oceanic ITCZ.

When the model is pushed toward Floquet unstable one-day periodic solutions, by allowing high values of the congestus adjustment parameter $\alpha_c$, for example, it exhibits multi-day periodic and chaotic solutions that are Floquet stable. Interestingly, as seen on Figure 9, those multi-day and chaotic solutions look very similar to the one-periodic solutions, characterized by an afternoon congestus heating peak that displays important day to day
variability. Because of the lack of observational evidence, it is not clear at this point, whether such solutions are realistic.

Acknowledgments.

The research of B. K. is supported by a grant from the Natural Sciences and Engineering Research Council of Canada and the Canadian Foundation for Climate and Atmospheric Sciences. The research of A. J. M. is supported by the National Science Foundation (NSF) Grant DMS-0456713, the office of Naval Research (NR) Grant N00014-05-1-0164, and the Defense Advanced Projects Agency Grant N0014-07-1-0750. Y. F. is a postdoctoral fellow supported through A.J.M’s above NSF and ONR grants. This research is partly achieved when B.K was visiting Courant Institute during his sabbatical year, 2009-2010, and completed at the Institute for Pure and Applied Mathematics when both Y. F and B.K were taking part as long-term visitors in the context of the long program on Climate Models and Data Hierarchies, for which A.J.M was one of the main organizers.
Floquet theory and numerical approximation of nonlinear periodic solutions

Here we give a brief outline of the mathematical methodology used here to find periodic solutions for the Galilean transformed systems in (4) and (5). It consists in searching numerically for periodic solutions in a given parameter regime. Once a periodic solution is found we verify its stability properties as a periodic orbit for the underlying dynamical system by applying the Floquet theory. For the more complex WK model (5), the Floquet theory is not practical, thus, to verify the stability of periodic solutions for this system we simply integrate the differential equations system forward in time with the initial condition provided by our periodic solution at the left end of the interval. We say that the periodic solution is stable if it resists the small numerical errors and that the integrated solution matches with good accuracy the reference solution over a few periods of integration.

a. Numerical approximation of nonlinear periodic solutions

To compute periodic solutions for the KM system in (4), we use a more or less standard routine found in the commercial software MATLAB, which is a uniformly fourth order accurate collocation numerical scheme (BVP4c). However, such standard routines are not
suitable for the WK system (5) since it cannot be written in an explicit form \( y' = f(y, t) \). Therefore, we design a simple algorithm for finding periodic solutions for this system using a second order one step finite difference method combined with the trapezoidal rule. More details for solving boundary value problems using such techniques can be found in (Ascher 1995). The same algorithm is used to evolve the solutions to demonstrate their dynamical stability for the WK system.

We consider the WK system written in an abstract vectorial form with periodic boundary conditions on one period-length interval \([0, T]\).

\[
y' = f(t, y, y'), \quad y(0) = y(T) \tag{A1}
\]

Given a grid of \( N + 1 \) points, \( t_i = ih \) where \( h = P/N \) is the mesh size, we approximate the derivative on the righthand side by finite differences and use the trapezoidal rule to approximate the differential equation. The resulting difference scheme is thus viewed as a system of nonlinear algebraic equations on the form

\[
\frac{y_{i+1} - y_i}{h} = \frac{1}{2} \left( f(t_i, y_i, \frac{y_{i+1} - y_i}{h}) + f(t_{i+1}, y_{i+1}, \frac{y_{i+2} - y_{i+1}}{h}) \right), 1 \leq i \leq N. \tag{A2}
\]

To enforce the periodicity of the solution, we set \( y_{N+1} = y_1 \) and \( y_{N+2} = y_2 \) to obtain an \( 11 \times N \) system which is solved via the routine \texttt{fsolve} of MATLAB.

While the above algorithm is used to search for periodic solutions, a simpler Euler-type difference scheme (A3) is used to solve the associated initial value problem, to evolve the system forward in time. At each time step, given \( y_i \) at time \( t_i \), we obtain \( y_{i+1} \) at \( t_{i+1} \) by
solving the system of 11 nonlinear algebraic equations

\[ y_{i+1} = hy_i + hf(t_i, y_i, \frac{y_{i+1} - y_i}{h}) \]  

(A3)

using the same \textit{fsolve} MATLAB routine but evolved over several periods of time to check for stability of a readily computed periodic solution as described above.

\textit{b. Floquet theory for non-linear periodic solutions}

Floquet theory is a standard tool for analysis of stability and bifurcations of periodic solutions of ordinary differential equations. A general overview of basic Floquet theory can be found in many elementary ODE textbooks such as (Hochstadt 1975) and a comprehensive reference for physical application of Floquet theory can be found in Dijkstra (2005). In this paper, we briefly review elementary Floquet theory as it is applied to our problem.

We consider the generic ODE system in vector form

\[ u' = f(u, t, \lambda_0) \]  

(A4)

where \( u \) is the (vector) dependent variable, \( t \) is time and \( \lambda_0 \) is a parameter. Let \( u_p(t) \) be a periodic solution of (A4), for a fixed value of \( \lambda_0 \), such that \( u_p(0) = u_p(T) \) for some period \( T > 0 \). The first order variation equation for a perturbation \( y(t) \), around periodic solution \( u_p(t) \), takes the form

\[ y' = A(t)y, \]  

(A5)

\[ y(0) = y_0, \]

where \( A(t) = \nabla_u f(u_p(t), t, \lambda_0) \). Here \( \nabla_u f \) is the Jacobian matrix derivative of the vector function \( f(u, t, \lambda_0) \) with respect to the vector \( u \). Let \( \mathcal{R} \) be a fundamental matrix of (A5),
i.e, \( \mathcal{R} \) has full rank and satisfies \( \mathcal{R}' = A(t)\mathcal{R}, \mathcal{R}(0) = I \). For a \( T \) periodic matrix \( A(t) \), the Floquet theorem guaranties that there exists a \( T \) periodic matrix \( \mathcal{P}(t) \) and a constant matrix \( \mathcal{F} \) such that \( \mathcal{R}(t) = \mathcal{P}(t)e^{\mathcal{F}t} \). Since \( \mathcal{P}(t) \) is periodic, growth or decay of perturbations over a period of time is determined by the eigenvalues of the *monodromy* matrix \( e^{\mathcal{F}T} \), know as the Floquet multipliers. It follows that if the magnitude of the largest eigenvalue is less/greater than one, the background periodic solution of system (A5) is asymptotically stable/unstable.

It is trivial to show that \( \mathcal{R}(T) = \mathcal{P}(T)e^{\mathcal{F}T} = e^{\mathcal{F}T} \) and therefore, in practice, we only need to find the fundamental solution and evaluate it at time \( T \). Direct integration of of (A5) via second order implicit midpoint scheme proves to be sufficient to compute the fundamental matrix at time \( t = T \).

Additionally, Floquet multipliers can be used to gain insight into the bifurcation of periodic solutions (Hochstadt 1975). Let \( p(\lambda) \) denote the largest (in magnitude) Floquet multiplier of the periodic solution \( u_p \) with parameter \( \lambda_0 = \lambda \). For real \( p(\lambda) \), \( p(\lambda) > 1 \) corresponds to cyclic-fold bifurcation while \( p(\lambda) < -1 \) is indicative of period doubling. Floquet multipliers crossing the unit circle through the complex plane corresponds to torus bifurcation. In fact, the bifurcation to chaos reported in section 4.c, corresponds to a case where the largest Floquet multiplier crosses the unit circle at minus one as the parameter \( \alpha_c \) is increased. Interestingly, the solution evolves from a stable one-day periodic solution to a two-day periodic solution and then to a four day-periodic solution to ultimately reach a chaotic behavior.
REFERENCES


Nesbitt, S. W. and E. J. Zipser, 2003: The diurnal cycle of rainfall and convective intensity according to three years of trmm measurements. *Journal of Climate*, 16 (10), 1456–1475.


List of Tables

1 Prognostic and diagnostic equations. See text and Table 2 for details. 41
2 Constants and parameters for the WK and KM models. 43
<table>
<thead>
<tr>
<th>Name</th>
<th>MK Model</th>
<th>WK model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum, 0th mode</td>
<td>None</td>
<td>[ \frac{\partial \theta_1}{\partial t} - \partial_x u_1 = H_d + \xi_s H_s + \xi_c H_c + S_1 ]</td>
</tr>
<tr>
<td>Momentum, ( j )st mode, ( j = 1, 2 )</td>
<td>[ \frac{\partial u_j}{\partial t} - \partial_x \theta_j = -C_d u_0 u_j - \frac{1}{\tau_R} u_j ]</td>
<td>[ \frac{\partial u_j}{\partial t} - \partial_x \theta_j + \partial_x (\tilde{u} u_j) = \frac{\sqrt{2}}{\tau_T} \delta \triangle_t u - \frac{1}{\tau_R} u_j ]</td>
</tr>
<tr>
<td>Potential temperature, 1st mode</td>
<td>[ \partial \theta_2 - \frac{1}{4} \partial_x u_1 = H_c - H_s + S_2 ]</td>
<td>[ \frac{\partial \theta_2}{\partial t} - \frac{\partial}{4} \partial_x u_1 + \tilde{u} \partial_x \theta_2 + \frac{\sqrt{2}}{4} \partial_x \tilde{u} = \frac{\partial}{H_c - H_s + S_2} ]</td>
</tr>
<tr>
<td>Potential temperature, 2nd mode</td>
<td></td>
<td>same</td>
</tr>
<tr>
<td>Radiative cooling</td>
<td>[ S_i = -Q_{i,R,i} - \tau_D^{-1} \theta_i ]</td>
<td>[ \frac{\partial q}{\partial t} + \tilde{u} \tilde{q} + \tilde{u} \theta [(u_1 + \tilde{u} u_2) q + \tilde{Q} (u_1 + \tilde{\lambda} u_2)] = -P + \frac{D}{\tau_T} ]</td>
</tr>
<tr>
<td>Free tropospheric moisture</td>
<td></td>
<td>[ \frac{\partial q}{\partial t} + \tilde{u} \tilde{q} + \tilde{u} [(u_1 + \delta u) q + (u_1 + \lambda u_2) \tilde{Q} - \tilde{u} \tilde{Q}_0] = -2\sqrt{2} \tau_T (H_d + \xi_s H_s + \xi_c H_c) + \frac{\tilde{E}}{\tau_T} \triangle t q + M_d + \partial_x \tilde{u} \triangle m q + (1 - \alpha_E) \frac{\tilde{E}}{\tau_T} \triangle t \theta + (1 - \beta_D) \frac{M_d}{\tau_T} + \partial_x \tilde{u} \triangle m \theta ]</td>
</tr>
</tbody>
</table>
Table 1 (Continued)

<table>
<thead>
<tr>
<th>Name</th>
<th>MK Model</th>
<th>WK model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary layer equivalent potential temperature</td>
<td>$\frac{\partial \theta_e}{\partial t} = \frac{1}{\tau_e} \Delta_s \theta_e - \frac{D}{h_b}$</td>
<td>None</td>
</tr>
<tr>
<td>Boundary layer velocity</td>
<td>$\frac{\partial u_b}{\partial t} + \bar{u} \partial_x \bar{u} = -\partial_x p_0 - \partial_x \bar{u}(\bar{u} - u_b) + \frac{E_u}{H_T} \triangle_t u$</td>
<td>$\partial u_b$ + $u_b \partial_x \theta_b = -\frac{E}{h_b} \triangle_t \theta - \frac{M_d}{h_b} \Delta_m \theta + \frac{1}{\tau_e} \Delta_s \theta - Q_{Rb}$</td>
</tr>
<tr>
<td>Boundary layer potential temperature</td>
<td>$\frac{\partial \theta_b}{\partial t} + u_b \partial_x \theta_b = -\frac{E}{h_b} \triangle_t \theta - \frac{M_d}{h_b} \Delta_m \theta + \frac{1}{\tau_e} \Delta_s \theta - Q_{Rb}$</td>
<td>$\frac{\partial q_b}{\partial t} + u_b \partial_x q_b = -\frac{E}{h_b} \triangle_t q - \frac{M_d}{h_b} \Delta_m q + \frac{1}{\tau_e} \Delta_s q$</td>
</tr>
<tr>
<td>Boundary layer moisture</td>
<td>None</td>
<td>$\frac{\partial H_s}{\partial t} = \frac{1}{\tau_s} \alpha_s H_d - H_s$</td>
</tr>
<tr>
<td>Stratiform heating</td>
<td>$\frac{\partial H_s}{\partial t} = \frac{1}{\tau_s} (\alpha_s H_d - H_s)$</td>
<td>Same</td>
</tr>
<tr>
<td>Congestus heating</td>
<td>$\frac{\partial H_c}{\partial t} = \frac{1}{\tau_c} (\alpha_c \Lambda Q_c^+ - H_c)$</td>
<td>Same</td>
</tr>
<tr>
<td>Deep convection</td>
<td>$H_d = (1 - \Lambda) Q_d^+$</td>
<td>Same</td>
</tr>
<tr>
<td>Maximum energy available for deep convection</td>
<td>$Q_d = \bar{Q} + \tau_{conv}^{-1} [a_1 \theta_e + a_2 q - a_0 (\theta_1 + \gamma_2 \theta_2)]^+$</td>
<td>Same</td>
</tr>
<tr>
<td>Maximum energy available for congestus convection</td>
<td>$Q_c = \bar{Q} + \tau_{conv}^{-1} [\theta_e - a_0' (\theta_1 + \gamma_2' \theta_2)]^+$</td>
<td>Same</td>
</tr>
<tr>
<td>Downdrafts</td>
<td>$D_c = m_0 [1 + \mu (H_s - H_c)/Q_{R,1}^0]^+ \Delta_m \theta_e$</td>
<td>$M_d = (D_c + h_b \partial_x u_b)^+$</td>
</tr>
<tr>
<td>Scalar entrainment velocity</td>
<td>None</td>
<td>$E = (M_u - M_d + h_b \partial_x u_b)^+$</td>
</tr>
<tr>
<td>Momentum entrainment velocity</td>
<td>None</td>
<td>$E_u = (\frac{h_b}{\tau_e} + h_b \partial_x u_b)^+$</td>
</tr>
</tbody>
</table>
Table 2. Constants and parameters for the WK and KM models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_b$ / $H_T$ / $\delta$</td>
<td>500 m / 16 km / 0.03125</td>
<td>ABL depth / Free troposphere depth / ratio of $h_b$ to $H_T$</td>
</tr>
<tr>
<td>$Q_{R1}$</td>
<td>1 K/day</td>
<td>First baroclinic radiative cooling rate</td>
</tr>
<tr>
<td>$Q_{R2}$</td>
<td>Determined at RCE</td>
<td>Second baroclinic radiative cooling rate</td>
</tr>
<tr>
<td>$Q_{Rb}$</td>
<td>Determined By RCE</td>
<td>Boundary layer radiative cooling rate</td>
</tr>
<tr>
<td>$\xi_s$ / $\xi_c$</td>
<td>0.5 / 1.25</td>
<td>Stratiform / Congestus contribution to first baroclinic mode</td>
</tr>
<tr>
<td>$\tilde{Q}$ / $\tilde{Q}_0$</td>
<td>0.9 / 6.5</td>
<td>Background moisture stratification / contribution to barotropic vertical moisture advection</td>
</tr>
<tr>
<td>$\tilde{\lambda}$ / $\tilde{\alpha}$</td>
<td>0.8 / 0.1</td>
<td>Coefficient of $u_2$ in linear / nonlinear moisture convergence</td>
</tr>
<tr>
<td>$m_0$</td>
<td>Determined at RCE</td>
<td>Large-scale background downdraft velocity</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.25</td>
<td>Contribution of convective downdrafts to $M_d$</td>
</tr>
<tr>
<td>$\alpha_s$ / $\alpha_c$</td>
<td>0.25 / 0.1</td>
<td>Stratiform / Congestus adjustment coefficient</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>0.2</td>
<td>Ratio of downdraft velocity ($M_d$) to upward mass flux velocity ($M_u$) at top of ABL</td>
</tr>
<tr>
<td>$\tau_R$ / $\tau_D$</td>
<td>75 days / 50 days</td>
<td>Rayleigh drag / Newtonian cooling time scale</td>
</tr>
<tr>
<td>$\tau_s$ / $\tau_c$</td>
<td>3 hours / 1 hour</td>
<td>Stratiform / Congestus adjustment time scale</td>
</tr>
<tr>
<td>$\tau_{conv}$</td>
<td>2 hours</td>
<td>Convective time scale</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>Determined by RCE</td>
<td>Surface evaporation time scale</td>
</tr>
<tr>
<td>$\tau_T$</td>
<td>8 hours</td>
<td>Momentum entrainment time scale</td>
</tr>
<tr>
<td>$\tilde{Q}$</td>
<td>Determined at RCE</td>
<td>Bulk convective heating at RCE</td>
</tr>
<tr>
<td>$a_1$ / $a_2$</td>
<td>0.45 / 0.55</td>
<td>Relative contribution of $\theta_{eb}$ / $q$ to deep convection</td>
</tr>
<tr>
<td>$a_0$ / $a_0'$</td>
<td>7 / 1.5</td>
<td>Dry convective buoyancy freq. in deep / congestus eq.</td>
</tr>
<tr>
<td>$\gamma_2$ / $\gamma_2'$</td>
<td>0.1 / 2</td>
<td>Relative contribution of $\theta_2$ to deep / congestus heating</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.1</td>
<td>Relative contribution of $\theta_2$ to $\theta_{em}$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>2</td>
<td>Ratio of $q_t$ to $q$</td>
</tr>
<tr>
<td>$C_d$</td>
<td>0.001</td>
<td>Surface drag coefficient</td>
</tr>
<tr>
<td>$u_0$</td>
<td>2 m/s</td>
<td>Strength of turbulent fluctuations</td>
</tr>
<tr>
<td>$\theta^-$ / $\theta^+$</td>
<td>10 K / 20 K</td>
<td>Moisture switch threshold values</td>
</tr>
</tbody>
</table>
1 Linear stability analysis of the homogeneous background RCE state of the KM model as a function of $\theta_e^* - \bar{\theta}_e$ and the stratiform rain fraction $f_s$ (top) and as a function of $\theta_e^* - \bar{\theta}_e$ and the congestus adjustment coefficient $\alpha_c$ (middle), for different thermodynamic backgrounds as indicated by $\bar{\theta}_e - \bar{\theta}_m = 12, 16, 14, 16$ K. The region of instability is shaded. The bottom plots show the largest Floquet multiplier of the one-periodic solution corresponding to the diurnal forcing in (3) as a function of $\alpha_c$. Note that although there a large region of instability of the homogeneous state on the middle-right panel, the bifurcation to instability of the periodic solution occurs only for roughly $\alpha_c \geq 0.33$. The rest of the parameters are as in Table 2.

2 Floquet stable periodic response of the KM model to a diurnal cycle forcing for the case of a moist background corresponding to $\bar{\theta}_e - \bar{\theta}_m = 12$ K. The rest of the parameters are fixed to their standard values in Table 2. The imposed perturbation of saturation equivalent potential temperature is given by the dashed line on the bottom right corner (denoted here by $\theta_e^{*,*}$).

3 Same as Fig. 2 but for the WK model.

4 Same as Fig. 2 but for $\bar{\theta}_e - \bar{\theta}_m = 16$ K.

5 Same as Fig. 4 but for the WK model.

6 Precipitation diagram for KM model (top) and MK model (bottom) under variation of atmospheric dryness. Standard values of parameters (taken from Table 1) are used for both models.
Precipitation diagrams for different boundary layer $\theta_e$ relative contributions to deep convection (from top to bottom: $a_1 = 0, 0.1, 0.45, 0.9, 1$) as a function of $\bar{\theta}_{eb} - \bar{\theta}_{em}$. The rest of the parameters are as in Table 2.

Contours of deep convective precipitation in the $(\bar{\theta}_{eb} - \bar{\theta}_{em})-(\theta_{eb}^* - \bar{\theta}_{eb})$ plan for different stratiform rain fractions. The rest of the parameters are as in Table 2. (Note that unlike Fig. 7, the contribution of stratiform and congestus precipitation are not included in these plots). All but purely congestus $(\bar{\theta}_{eb} - \bar{\theta}_{em} = 20)$ states are Floquet stable.

Chaotic solution evolving from a one-day Floquet unstable solution. The dashed lines represent the Floquet unstable one-day periodic solution: $\alpha_c = 0.4$ and $\bar{\theta}_{eb} - \bar{\theta}_{em} = 14$ K. The rest of the parameter are as in Table 2.
Fig. 1. Linear stability analysis of the homogeneous background RCE state of the KM model as a function of $\theta_{eb}^* - \bar{\theta}_{eb}$ and the stratiform rain fraction $f_s$ (top) and as a function of $\theta_{eb}^* - \bar{\theta}_{eb}$ and the congestus adjustment coefficient $\alpha_c$ (middle), for different thermodynamic backgrounds as indicated by $\bar{\theta}_{eb} - \bar{\theta}_{em} = 12, 16, 14, 16$ K. The region of instability is shaded. The bottom plots show the largest Floquet multiplier of the one-periodic solution corresponding to the diurnal forcing in (3) as a function of $\alpha_c$. Note that although there a large region of instability of the homogeneous state on the middle-right panel, the bifurcation to instability of the periodic solution occurs only for roughly $\alpha_c \geq 0.33$. The rest of the parameters are as in Table 2.
Fig. 2. Floquet stable periodic response of the KM model to a diurnal cycle forcing for the case of a moist background corresponding to $\bar{\theta}_{eb} - \bar{\theta}_{em} = 12$ K. The rest of the parameters are fixed to their standard values in Table 2. The imposed perturbation of saturation equivalent potential temperature is given by the dashed line on the bottom right corner (denoted here by $\theta^{*'}_{eb}$).
Fig. 3. Same as Fig. 2 but for the WK model.
\[ \theta_{eb} - \theta_{em} = 16, \quad f_s = 0.4, \quad f_c = 0.1, \quad \alpha_c = 0.1, \quad a_1 = 0.45, \quad a_2 = 0.55 \]

**Fig. 4.** Same as Fig. 2 but for $\bar{\theta}_{eb} - \bar{\theta}_{em} = 16$ K.
Fig. 5. Same as Fig. 4 but for the WK model.
Fig. 6. Precipitation diagram for KM model (top) and MK model (bottom) under variation of atmospheric dryness. Standard values of parameters (taken from Table 1) are used for both models.
Fig. 7. Precipitation diagrams for different boundary layer $\theta_e$ relative contributions to deep convection (from top to bottom: $a_1 = 0, 0.1, 0.45, 0.9, 1$) as a function of $\bar{\theta}_{eb} - \bar{\theta}_{em}$. The rest of the parameters are as in Table 2.
Fig. 8. Contours of deep convective precipitation in the $(\bar{\theta}_{eb} - \bar{\theta}_{em})$–$(\theta^*_eb - \bar{\theta}_{eb})$ plan for different stratiform rain fractions. The rest of the parameters are as in Table 2. (Note that unlike Fig. 7, the contribution of stratiform and congestus precipitation are not included in these plots). All but purely congestus ($\bar{\theta}_{eb} - \bar{\theta}_{em} = 20$) states are Floquet stable.
Fig. 9. Chaotic solution evolving from a one-day Floquet unstable solution. The dashed lines represent the Floquet unstable one-day periodic solution: $\alpha_c = 0.4$ and $\bar{\theta}_{eb} - \bar{\theta}_{em} = 14$ K. The rest of the parameter are as in Table 2.