Partial Differential Equations for Finance
MATH-GA 2706, Spring 2015
Mondays 5:10-7pm in WWH 512

Modified 2/1/2015 – grader info added, final exam date changed
Further modification 2/3: Samu Alanko's office hours added

Instructor: Robert V. Kohn. Office: 502 WWH. Phone: 212-998-3217. Email: kohn@cims.nyu.edu. Web: math.nyu.edu/faculty/kohn. Office hours (tentative): Mon 2-3 and Fri 4:30-5:30

Grader: Samu Alanko. Office: WWH 1003. Email: alanko@cims.nyu.edu. Office hour: Wed 5-7.

Prerequisites: Working knowledge of stochastic calculus, and basic familiarity with the Black-Scholes approach to option pricing.

Content: An introduction to those aspects of partial differential equations and optimal control most relevant to finance. PDE's naturally associated to diffusion processes: the forward and backward Kolmogorov equations and their applications. Linear parabolic equations: fundamental solution, boundary value problems, maximum principle. Dynamic programming and optimal control: Hamilton-Jacobi-Bellman equation, verification arguments, optimal stopping. Applications to finance will be distributed throughout the course, including: barrier options; options on an underlying that can jump; portfolio optimization; American options; and other examples of optimal decision-making.

Comparison to the class Applied Stochastic Analysis: PDE for Finance is part of the Math Finance MS curriculum; while well-prepared students from all programs are welcome, the course's content is chosen with Math Finance MS students in mind. In particular: PDE for Finance assumes a working knowledge of stochastic calculus and the Black-Scholes approach to option pricing, but it does not assume prior exposure to PDE. Almost half the semester is devoted to portfolio optimization, optimal stopping, and other problems from stochastic optimal control. By contrast, Applied Stochastic Analysis (MATH-GA 2704) was designed mainly with early-stage Mathematics PhD students in mind. It therefore assumes a broad mathematical background (including some ODE and PDE) and a lot of scientific maturity, though it does not assume prior experience with stochastic calculus. Applied Stochastic Analysis emphasizes applications in the physical sciences, and spends little or no time on stochastic control.

Special Dates: Since NYU was closed on 1/26, the first lecture is 2/2. No lecture 2/16 (Presidents' Day) and 3/16 (Spring Break). The final exam will be Monday 5/18, in our normal class slot (5:10-7pm, WWH 512). *This is a change from the preliminary syllabus posted over Xmas break.*

Homeworks, exams, grades: There will be 6 or 7 homework assignments, and a final exam. Grades will be based on the HW (1/2) and the final (1/2). Collaboration on HW is encouraged (homeworks are not exams), but students must write up and turn in their solutions individually. If you work closely with another student (or someone else, or you use other resources such as something on the web) please identify your collaborators and/or sources on your solution sheet. HW may be turned in late only by securing permission before it is due; no credit will be given for HW turned in after a solution sheet has been distributed. The HW is important not only because it counts as part of the grade, but also because it helps you master the material (and therefore helps you do well on the exam). The final will be closed-book, but you may bring two 8.5×11

sheets of notes (both sides, any font). Requests to take a makeup exam must be made in advance, and will not be granted for matters of personal convenience.

Resources: Lecture notes and homework sets will be posted on my website as they become available. An NYU-Classes site will be used for announcements, and for material I don't want to make public (such as HW solutions). Auditors can be added to the NYU-Classes site upon request. See a separate handout for a list of books that correlate with various parts of this class.

Semester plan: Lectures 1-2: the backward and forward Kolmogorov equations and their applications. Lectures 3-4: the linear heat equation – properties and solution formulas on the real line, a half-line, and an interval. Lectures 5-6: deterministic optimal control. Lectures 7-9: stochastic optimal control, including portfolio optimization, optimal stopping, and American options. Lectures 10-13: additional topics – some possibilities include pricing options on underlyings with jumps, the martingale approach to portfolio optimization, and asset price bubbles from heterogeneous expectations.