Derivative Securities, Fall 2012 – Homework 6. Distributed Thurs 11/29, due by 5pm Fri 12/14, NO EXTENSIONS.

Please note:

- Our last class is Wed 12/5. (Wed 12/12 is a "Legislative Day" at NYU; classes meet on a Monday schedule.)
- You may turn in HW6 by (a) putting hard copy in my WWH Lobby mailbox (alphabetical among the faculty mailboxes, it's near the middle of the wall opposite the window), (b) slipping hard copy under my office door (WWH 502), (c) giving it to Xingxin or me in person, or (d) sending electronic copy to Xingxin (zhong@cims.nyu.edu). No extensions will be granted, because I want to distribute a solution sheet well before the exam.
- As previously announced, our exam is Wed 12/19, in the normal class location and timeslot. You may bring two sheets of notes (8.5×11, both sides, any font). No other notes, calculators, or other electronic devices will be permitted.
- The material we covered in class on 11/28 is fair game for the exam. This means everything in the Section 11 notes *except* the discussion of the CreditGrades model. The material we cover in class on 12/5 (corresponding to the not-yet-distributed Section 12 notes) will not be on the exam.

Problem 1 provides additional practice with the pricing of swaptions and caplets using Black's formula. Problem 2 provides practice in the pricing of defaultable bonds and CDS's. Problem 3 asks you to find (risk-neutral) default probabilities from market prices. Problem 4 reinforces our discussion of the Merton model.

(1) Suppose the risk-free discount factors are as follows:

$1Y$	.9450
$2Y$	.8900
$3Y$	.8250
$4Y$	.7550
$5Y$	.6700

Ignore any possibility of default.

(a) Calculate the par swap rate of a forward-starting swap that starts at the end of year 2 and pays annual coupons for three years (so that the coupon payments are at the ends of years 3, 4, and 5).

(b) Using the result of part (a), calculate the value of a swaption on a 3 year annual payment swap to receive the floating rate and pay a fixed rate of 6.50% that is exercisable in 2 years (if the swap is exercised, it has 3 years to run from the exercise date). Value the swaption based on an annual interest rate volatility of 15.0%

(c) Calculate the value of a caplet on the 1 year LIBOR rate 3 years from now, with a cap rate of 6.50% and an annual interest rate volatility of 18.0%.

(2) Suppose the 1Y-5Y risk-free discount factors are as given in Problem 1, and suppose the risk-free discount factors for half-year maturities are:

$0.5Y$	.9650
$1.5Y$	.9200
$2.5Y$	.8550
$3.5Y$	.7850
$4.5Y$	.7100.

Suppose further that the conditional probabilities of default (i.e. probabilities of default, given survival to that year) for a particular corporation are:

$1Y$	2.00%
$2Y$	2.50%
$3Y$	3.00%
$4Y$	3.50%
$5Y$	4.00%

and assume a recovery rate in event of default of 25%.

(a) Calculate the par CDS spread for a 5 year CDS with annual swap payments. What would be the value to the protection provider of a 5 year CDS with an annual swap rate of 1.75%?

(b) Calculate the par coupon rate for a 5 year bond issued by this corporation with annual coupon payments. (By definition: the par coupon rate is the one that makes the value of the bond equal to its principal. You should allow for the possibility of default in doing this calculation). What is the value of a bond with an annual coupon of 8.00%?

(3) Suppose the risk-free yield curve is flat at 6% with annual compounding. One-year, two-year, and three-year bonds yield 7.2%, 7.4%, and 7.6% with annual compounding. All have 6% coupons with annual payments, and all are issued by the same corporation. Assume that in case of default the recovery is 40% of principal, paid at the end of the year; also, assume that in case of default there is no payment of accrued interest. Find the risk-neutral probability p_i that default occurs during year i, for $i = 1, 2, 3$. [In defining the yield of a bond one normally ignores the possibility of default; thus, for example, the yield y of the two-year bond considered here is related to its price by

$$\text{price} = \frac{c}{1+y} + \frac{c}{(1+y)^2} + \frac{P}{(1+y)^2}$$

where c is the coupon payment and P is the principal.]

(4) This is a problem on implementing the Merton model. Assume that a company has a current stock price of 22.30, and outstanding debt per share of 30, all of which matures in 5 years and pays no coupon. Assume further that the current risk-free rate is 5.5% (with continuous compounding) at all maturities, and that the equity volatility is 30%. Finally, assume that the value of the firm is 45.00 and that the volatility of firm value is 15%.

(a) Does the value of $V_0 N(d_1) - e^{-rT} D N(d_2)$ match the value of E_0?

(b) What is the probability of default, according to the Merton model?

(c) Does the value of $(\sigma_E E_0)/(\sigma_V V_0)$ match the value of $N(d_1)$?

(d) Has the Merton model been correctly calibrated?