**Derivative Securities, Fall 2012 – Homework 5. Distributed 11/15, due 11/28.**

(1) Suppose the LIBOR discount rates $B(0,t)$ are given by the table below. Consider a 3-year swap whose floating payments are at the then-current LIBOR rate, and whose fixed payments are at the term rate of $R_{\text{fix}}$ per annum.

| payment date $t_i$ | $B(0,t_i)$ |
|---|---|
| 0.5 | .9748 |
| 1.0 | .9492 |
| 1.5 | .9227 |
| 2.0 | .8960 |
| 2.5 | .8647 |
| 3.0 | .8413 |

(a) Suppose $R_{\text{fix}}$ is 6.5 percent per annum and the notional principal is 1 million dollars. What is the value of the swap?

(b) What is the par swap rate? (In other words: what value of $R_{\text{fix}}$ sets the value of the swap to 0?)

(2) You are given the following market quotes:

| | | |
|---|---|---|
| 3 month LIBOR | = | 4.55% |
| 1st 3 months forward term rate (starts in 3 months) | = | 5.00% |
| 2nd 3 months forward term rate (starts in 6 months) | = | 5.35% |
| 3rd 3 months forward term rate (starts in 9 months) | = | 5.75% |
| 1.5 year par swap rate | = | 6.50% |
| 2 year par swap rate | = | 6.70% |
| 2.5 year par swap rate | = | 6.90% |
| 3 year par swap rate | = | 7.00% |

Using bootstrapping, derive a set of semi-annual discount rates from these inputs.

(3) There are two ways to value a swap:

(i) We can view the swap as a collection of forward rate agreements, with payment dates $0 < t_1 < \ldots < t_N$ and rate $R_{\text{fix}}$. This approach gives

$$\text{swap value} = \sum_{i=1}^{N} B(0,t_i)[R_{\text{fix}} - f_0(t_{i-1},t_i)](t_i - t_{i-1})L$$

where $L$ is the notional principal, and $f_0(t_{i-1},t_i)$ is the forward term rate for lending from $t_{i-1}$ to $t_i$, defined by

$$f_0(t_{i-1},t_i)(t_i - t_{i-1}) = \frac{B(0,t_{i-1})}{B(0,t_i)} - 1.$$

(ii) We can view the swap as the difference between a fixed-rate bond and a floating-rate bond. With the same notation as above, this approach gives

$$\text{swap value} = \sum_{i=1}^{N} B(0,t_i)R_{\text{fix}}(t_i - t_{i-1})L - (1 - B(0,t_N))L.$$

Show that these two approaches are consistent, i.e. the swap values given in (i) and (ii) above are equal.

(4) [Hull, Chapter 28, problem 22, slightly modified.] Calculate the price of a cap on the three-month LIBOR rate in nine months' time when the principal amount is $1000. Use Black's model and the following information:

- The nine-month Eurodollar futures price is 92 (ignore the difference between forwards and futures).
- The interest rate volatility implied by a nine-month Eurodollar option is 15 percent per annum.
- The current 12-month interest rate with continuous compounding is 7.5 percent per annum.
- The cap rate is 8 percent per annum.

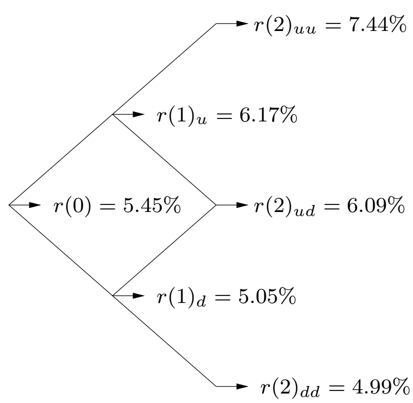
[Note: by market convention, the Eurodollar futures price refers to a 3-month contract; since we are ignoring the difference between forwards and futures, this amounts to a forward term rate. So the practical meaning of the first bullet is that we can secure, at no cost now, the right to a 3-month Eurodollar contract starting 9 months from now at 100-92=8 percent per annum (another market convention).]

(5) [Hull, Chapter 28, problem 23] Suppose the LIBOR yield curve is flat at 8% with annual compounding. Consider a swaption that gives its holder the right to receive 7.6% in a five-year swap starting in four years. Payments are made annually. The volatility for the swap rate is 25% per annum and the principal is $1 million.

(a) In using Black's model to value this instrument, which formula should be used (the one associated with a call, or the one associated with a put)?

(b) Price the swaption using Black's model.

(6) Consider the binomial tree of interest rates shown in the figure (each time interval is one year, and the rates shown are per annum with continuous compounding). Assume the risk-neutral probabilities are 1/2 for each branch.




(a) Find the values of $B(0,1)$, $B(0,2)$, and $B(0,3)$.

(b) Consider the following European call option written on a one year Treasury bill: its maturity is $T = 2$, and its strike is 0.945, so the payoff at time 2 is $(B(2,3) - 0.945)_+$. Find the value of this option at time 0.