Derivative Securities, Fall 2012 – Homework 1. Distributed at Lecture 1. Due in class at Lecture 3 (9/19/12).

1. The present exchange rate between US dollars and Euros is 1.34 \$/Euro. The price of a domestic 180-day Treasury bill is \$99.50 per \$100 face value. The price of the analogous Euro instrument is 98.50 Euros per 100 Euro face value.
   (a) What is the theoretical 180-day forward exchange rate?
   (b) Suppose the 180-day forward exchange rate available in the marketplace is 1.31 \$/Euro. This is less than the theoretical forward exchange rate, so an arbitrage is possible. Describe a risk-free strategy for making money in this market. How much does it gain (in dollars), for a contract size of 100 Euro?

2. Let $B(t,T)$ be the cost at time t of a risk-free dollar at time T.
   (a) Suppose $B(0,1)$, $B(0,2)$ and $B(1,2)$ are all known at time 0 (i.e. interest rates are deterministic). Show that the absence of arbitrage requires $B(0,1)B(1,2) = B(0,2)$.
   (b) Now suppose $B(0,1)$ and $B(0,2)$ are known at time 0 but $B(1,2)$ will not be known until time 1. What goes wrong with your argument for (a)? Show that if we know with certainty that $m \leq B(1,2) \leq M$ then we can still conclude $mB(0,1) \leq B(0,2) \leq MB(0,1)$.

3. The present price of a stock is 50. The market value of a European call with strike 47.5 and maturity 180 days is 4.375. The cost of a risk-free dollar 180 days hence is $B(0,180) = .99$.
   (a) For a European put with a strike price of 47.5 you are quoted a price of 1.325. Show this is inconsistent with put-call parity.
   (b) Describe how you can take advantage of this situation, by finding a combination of purchases and sales which provides an instant profit with no liability 180 days from now.

4. For each of the following portfolios, draw the expiry payoff diagram and explain what view of the market holding this position expresses:
   (a) long one call and one put, both with strike price K (this is known as a *straddle*);
   (b) short one forward and long two calls, all with strike price K (this is also a *straddle* – why?);
   (c) Long one call and two puts, all with strike price K (this is known as a *strip*);
   (d) Long one put and two calls, all with strike price K (this is known as a *strap*);
   (e) Long one call with strike K_2 and one put with strike K_1. Compare the cases $K_2 > K_1$ (known as a *strangle*), $K_2 = K_1$, and $K_2 < K_1$.
   (f) Long one call with strike K_1, long one call with strike K_2, and short two calls with strike $(K_1 + K_2)/2$ (this is known as a *butterfly spread*).

5. The interest rate is $r = 0$. You observe the following prices in the market (all options are on the same underlying, with the same maturity time T):

   - The stock trades at $S_0 = 100$.
   - A straddle with $K = 100$ trades at 10.0.
   - A strip with $K = 100$ trades at 15.0.
   - A strap with $K = 100$ trades at 15.0.
   - A strangle with $K_1 = 95$ and $K_2 = 105$ trades at 5
   - A butterfly spread with $K_1 = 95$ and $K_2 = 105$ trades at 2.

   Describe a risk-free strategy that involves trading only these instruments.

6. Use arbitrage arguments to prove the following bounds on the price $C(S_0, K, T)$ of a European call with strike K and maturity T (assuming the underlying pays no dividend):

   (a) The call price is no greater than the stock price: $C(S_0, K, T) \leq S_0$.

   (b) For otherwise identical calls with strikes $K_1 < K_2$,

   $$0 \leq C(S_0, K_1, T) - C(S_0, K_2, T) \leq K_2 - K_1.$$

   (c) For othwerwise identical calls with maturities $T_1 < T_2$,

   $$C(S_0, K, T_1) \leq C(S_0, K, T_2).$$

7. An investor holds a European call with strike K_c and maturity T on a non-dividend-paying asset whose current price is S_0. Suppose the investor can write a put with any strike strike price K_p, write a forward with any delivery price K_f, and can borrow any amount B at the risk-free rate (if B is negative this is a loan). What are the conditions on K_p, K_f, and B that make this combination of positions a constructive sale (i.e. that have the same effect as selling the call)?

8. Suppose current spot price of oil is \$95 per barrel, the six-month forward price of oil is \$94 per barrel, and the current risk-free rate for 6 months is 2.00% per annum. What effective six-month borrowing rate for oil does this imply? Describe the actions you would take to achieve this borrowing rate and demonstrate that these actions result in a borrowing at this rate.