Derivative Securities - Fall 2007 - Section 12
supplement to Steve Allen's notes

Recall notation + basics of large pool, homog., base correlation, single-factor, Gaussian copula model:

- focus on assessing losses due to default up to a fixed time t. Assume $D = D(t)$ = (prob that any single name has defaulted by time t) is known. Also, ρ = correlation is known (+ constant + uniform). Then by 1-factor hypothesis + Gaussian copula model, with M = the common factor,

  prob of default of i^{th} name by time t, given value of M is

$$N\left(\frac{N^{-1}(D) - \sqrt{\rho}\,M}{\sqrt{1-\rho}}\right) = \Omega_t(M)$$

- under "large pool" hypothesis we use law of large numbers to say Ω_t is the "exact" fraction of pool that has defaulted by time t.

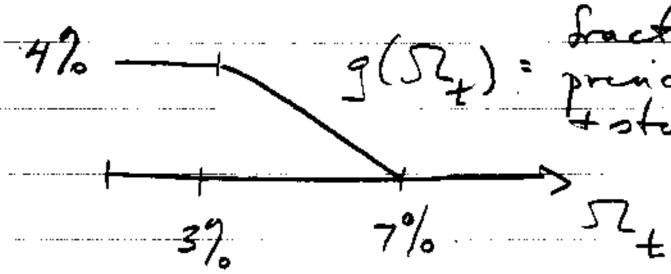
What to do with this? Well, for any single tranche of a CDO that's based on

CDS's (a "synthetic CDO") there are two cash flows

protection purchaser pays

$$N \frac{s}{f}\, g(\Omega_{t_j}) \qquad \text{at the } t_j$$

where s = spread, N = principal, f = freq, and $g(\Omega_t)$ is a piecewise-linear fn reflecting the attachment + detachment pts of tranche under consideration, eg for 3% to 7% tranche



$g(\Omega_t)$ = fraction of total principal assoc this tranche + still alive.

Value this payment by taking expected value + mult by discount factor

$$N \frac{s}{f} B(0, t_j)\ \mathbb{E}\left[g(\Omega_t(M)) \right]$$

where $\mathbb{E}$ = expectation wrto Gaussian statistics of M. Calculate expectation numerically, eg by Gaussian quadrature (or any numerical integn scheme).

<u>protection provider</u> pays

$$N(1-R)\left[-g(\Omega_{t_j}) + g(\Omega_{t_{j-1}})\right]$$

where recovery rate is R, &

$$-g(\Omega_{t_j}) + g(\Omega_{t_{j-1}}) = \underline{\text{fraction of total prin assoc this tranche that defaulted during most}} \text{ recent period}$$

This too is calculated numerically; since it is linear we need only calculate

$$\mathbb{E}\left[\, g(\Omega_t(M))\, \right]$$

at t_j + t_{j-1}. (Calculations differ only by having different values of $D = D(t_j)$.) Discount of course to get present value

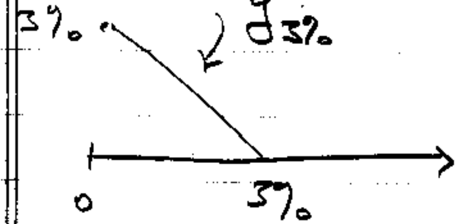
$$N(1-R)\, B(0,t_j) \quad \mathbb{E}\left[g(\Omega_{t_{j-1}}(M)) - g(\Omega_{t_j}(M))\right].$$

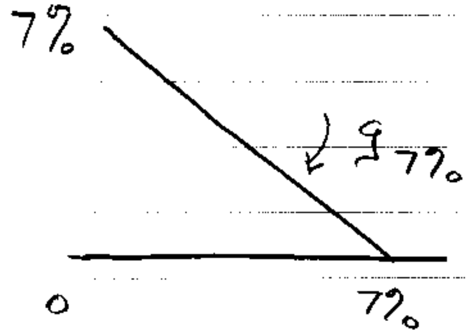
Steve's table (pg 5) demonstrates calcn of $\mathbb{E}[g(\Omega_t(M))]$ using crude Gaussian quadrature (4 quadr pts, so it's possible to do by hand) for two choices of tranche

① The 0% to 3% base tranche

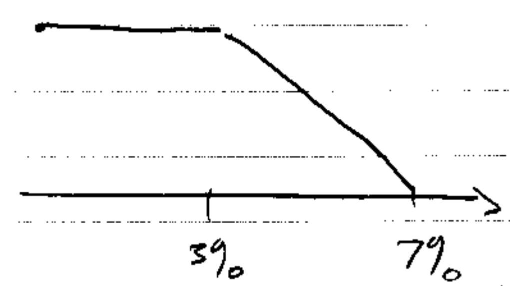
② the 0% to 7% base tranche

Note that for base tranches g looks like





so $g_{7\%} - g_{3\%}$ is



ie precisely the g assoc to the 3% to 7% tranche. Thus for computing expected losses we may always use base tranches then subtract. (Base tranches are more convenient for defining an implied correlation, since for base tranches the expected loss is monotone in ρ.)

Suggestion for further reading that's consistent with viewpoint discussed here:

"Credit Risk Models IV: Understanding + Pricing CDO's" by Abel Elizalde, working paper, available at www.abelelizalde.com