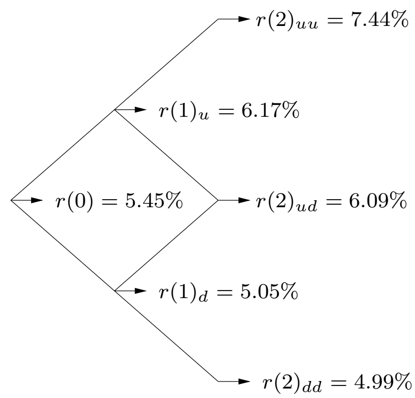
Derivative Securities, Fall 2007 – Homework 6. Distributed 11/14/07, due *by classtime* 12/12/07. Typo in problem 4c corrected 12/2/07.

Please note:

- This problem set includes a question on the Merton model, and one on CDO's. This material might not be covered in lecture until Wed 11/28.
- The solutions to this problem set will be discussed *in class* on Wed 12/12. Therefore you must hand in your homework prior to or at class on 12/12 to get credit. The solution sheet will be posted online 12/13.
- Our final exam is Wed 12/19, at the normal class meeting time and place. You may bring two 8.5×11 sheets of notes (both sides, any font), but no other books, notes, or calculators are permitted. Students taking the exam 5:10-7pm will not be permitted to leave early. If you'd like to swap timeslots (e.g. take the exam 7:10-9 though you're registered for 5:10-7) please ask permission in advance from your official instructor (kohn@cims.nyu.edu or allen@cims.nyu.edu).

1. Consider the binomial tree of interest rates shown in the figure (each time interval is one year, and the rates shown are per annum with continuous compounding). Assume the risk-neutral probabilities are 1/2 for each branch.



   (a) Find the values of $B(0,1)$, $B(0,2)$, and $B(0,3)$.

   (b) Consider the following European call option written on a one year Treasury bill: its maturity is $T = 2$, and its strike is 0.945, so the payoff at time 2 is $(B(2,3) - 0.945)_+$. Find the value of this option at time 0.

2. Suppose the credit-risk-free discount factors are as follows:

$1Y$	.9450
$2Y$	.8900
$3Y$	.8250
$4Y$	.7550
$5Y$	.6700

   (a) Calculate the par swap rate of a forward-starting swap that starts at the end of year 2 and pays annual coupons for three years (so that the coupon payments are at the ends of years 3, 4, and 5).

(b) Using the result of part (a), calculate the value of a swaption on a 3 year annual payment swap to receive the floating rate and pay a fixed rate of 6.50% that is exercisable in 2 years (if the swap is exercised, it has 3 years to run from the exercise date). Value the swaption based on an annual interest rate volatility of 15.0%

(c) Calculate the value of a caplet on the 1 year LIBOR rate 3 years from now, with a cap rate of 6.50% and an annual interest rate volatility of 18.0%.

3. Suppose the 1Y-5Y credit-risk-free discount factors are as given in Problem 2, and suppose the credit-risk-free discount factors for half-year maturities are:

$0.5Y$	.9650
$1.5Y$	.9200
$2.5Y$	.8550
$3.5Y$	.7850
$4.5Y$	.7100.

Suppose further that the conditional probabilities of default (i.e. probabilities of default, given survival to that year) are:

$1Y$	2.00%
$2Y$	2.50%
$3Y$	3.00%
$4Y$	3.50%
$5Y$	4.00%

and assume a recovery rate in event of default of 25%.

(a) Calculate the breakeven swap spread (also called the par CDS spread) for a 5 year CDS with annual swap payments. What would be the value to the protection provider of a 5 year CDS with an annual swap rate of 1.75%?

(b) Calculate the par coupon rate for a 5 year corporate bond with annual coupon payments. What would be the price if the bond had an annual coupon of 8.00%?

4. This is a problem on implementing the Merton model. Assume that a company has a current stock price of 22.30, and outstanding debt per share of 30, all of which matures in 5 years and pays no coupon. Assume further that the current risk-free rate is 5.5% (with continuous compounding) at all maturities, and that the equity volatility is 30%. Finally, assume that the value of the firm is 45.00 and that the volatility of firm value is 15%.

(a) What is the probability of default?

(b) Does the value of $(\sigma_E E_0)/(\sigma_V V_0)$ match the value of $N(d_1)$?

(c) Does the value of $V_0 N(d_1) - e^{-rT} D N(d_2)$ match the value of E_0?

5. Consider a portfolio of loans with average default probability of $D = 6.00\%$, recovery rate $R = 25\%$ and correlation $\rho = 40\%$. Using the large pool homogeneous base correlation single-factor Gaussian copula model, calculate the tranche losses for all of the CDX tranches: $0\% - 3\%, 3\% - 7\%, 7\% - 10\%, 10\% - 15\%$ and $15\% - 30\%$. In your numerical integration, use four equally spaced probabilities of $12.5\%, 37.5\%, 62.5\%$ and 87.5%, as in the example in the Section 12 notes.