Derivative Securities, Fall 2007 – Homework 2. Distributed 9/19/07, due 10/3/07.

Note: A solution sheet to HW1 will be posted 9/26; a solution sheet to HW2 will be posted 10/10; no late HW's will be accepted once the corresponding solution sheet has been posted.

Problems 1 and 2 reinforce our discussion of one-period markets (the Section 2 notes).

1. You suspect that Delta airlines will merge with Northwest Airlines in the coming month. Delta stock is trading at \$0.85. **We assume that the forward price equals the stock price.** There is a 60% chance that the merger will occur, in which case the stock will be worth \$1.20. There is a 40% chance that the merger will not occur, in which case the stock will continue its downward plunge to \$0.30. If $r = 0.02$ per annum, what is the value of a call with strike price $K = \$1.00$ and maturity one month (i.e. $T = 1/12$)? If you were to sell one thousand of these calls, explain precisely now much of the forward you would buy or sell in order to hedge.

2. Consider a one-period market in which the forward price is described by a trinomial tree: the forward price today is $F_0 = 100$; at the final time T the three possible states are $F_u = 120$, $F_m = 110$, and $F_d = 80$. Suppose the risk-free rate is $r = 0$. Consider an option whose payoff is $V_u = 30$ if the final-time state is F_u, $V_m = 20$ if the final-time state is F_m, and $V_d = 0$ if the final-time state is F_d. We're interested in the value V_0 of the option today.

   (a) Show that there is no real number ϕ such that $V_u + \phi F_u = V_m + \phi F_m = V_d + \phi F_d$. Conclude that V_0 cannot be determined by arbitrage pricing alone.

   (b) Consider changing the problem by altering the value of V_m (leaving all other numbers unchanged). For which choice(s) of V_m would the option's value V_0 be determined by arbitrage pricing alone?

   (c) We know (by an arbitrage argument, using only the option and a bond) that the value of the option is restricted by its largest and smallest payoff: $0 \leq V_0 \leq 30$. Use an arbitrage argument involving the forward to prove tighter bounds. Can you identify the tightest bounds achievable by such an argument?

Problems 3-5 reinforce our discussion of multiperiod binomial trees (the Section 3 notes). Each asks you to price or hedge an option on the forward price. We'll specify a binomial tree for the forward price by giving you F_{now}, u, and d, with the convention that $F_{\text{up}} = uF_{\text{now}}$ and $F_{\text{down}} = dF_{\text{now}}$. The risk-free rate r will always be constant in time.

3. A European put option with strike price 45 dollars matures in one year. The current forward price is 50 dollars, and the risk-free rate is 5 percent per annum. Divide the one-year interval into two six-month intervals, and use the recombinant tree with $u = 1.17$ and $d = .88$.

(a) Determine the put price by working backward through the tree.

(b) Determine the put price by using the formula which gives it as an average over all final-time payoffs. Of course your answer should be the same as for (a).

(c) Describe the associated trading strategy using forwards. In other words, specify how many units of the forward you should hold at each node (after rebalancing) to replicate the option payoff.

4. The current forward price is 100. The risk-free interest rate is 5 percent per annum. Consider a one-year European call option on this forward price with strike 100.

   (a) Divide the one-year period into two six-month intervals, and use the recombinant tree with $u = 1.25$ and $d = .82$. Calculate the risk-neutral probabilities. What is the option value?

   (b) Suppose the market price of the option is 16. Assuming the market is truly described by the tree of part (a), there must be an arbitrage. Explain in detail (specifying all trades) how you can take advantage of the "incorrect" market price to earn a risk-free profit.

5. A special kind of one-year put option is written on the forward price. The current forward price is 40 and the current strike price is 40. At month 6, if the forward price is below 35 the strike price is lowered to 35; otherwise it remains unchanged. The risk-free interest rate is 5 percent per annum.

   (a) Use a 2-period binomial tree with $u = 1.21$ and $d = .81$ to value the option.

   (b) Now use a 4-period binomial tree with $u = 1.1$ and $d = .9$ to value it.

   (c) What is the difficulty with valuing this type of option?