The length PA, PB, and PC are $|1-z|$, $|\omega-z|$, and $|\omega^2-z|$. Here we use complex numbers to represent points. z is the point P and A, B and C are the complex numbers 1, ω, and ω^2. These are the cube roots of 1, and $\omega = \dfrac{1+i\sqrt{3}}{2}$, $\omega^2 = \dfrac{1-i\sqrt{3}}{2}$. As in the lecture, the complex numbers $1-z$, $\omega-z$, and ω^2-z do not sum to 0. (If they did, their lengths would be the sides of a triangle.) Instead, we multiply these by 1, ω and ω^2 respectively to get the complex numbers

$$1-z,\ \omega(\omega-z),\ \omega^2(\omega^2-z) \tag{1}$$

These do sum to 0, and have the same lengths as PA, PB, and PC, and so these lengths do for the sides of a triangle. It is only necessary to show that the area of this triangle depends only on $|z|$. Here we use the fact that the area of triangle OST in the plane, with $O=(0,0)$, $S=(a,b)$, and $T=(c,d)$, is

$$K = \frac{1}{2}|ad-bc|$$

We use complex numbers to compute this. Take $z=a+bi$ and $w=c+di$. Then

$$\overline{z}w = (a-bi)(c+di) = (ac+bd)+(ad-bc)i$$

Take conjugates to get

$$z\overline{w} = (ac+bd)-(ad-bc)i$$

Subtract and divide by $2i$ to get

$$|ad-bc| = \left|\frac{\overline{z}w - z\overline{w}}{2i}\right| = \frac{1}{2}|\overline{z}w - z\overline{w}|.$$

So

$$K = \frac{1}{4}|\overline{z}w - z\overline{w}|.$$

In the problem at hand, we replace z by $(1-z)$ and w by $\omega(\omega-z) = \omega^2 - \omega z$. This gives (using $\overline{\omega} = \omega^2$ and $\overline{\omega^2} = \omega$)

$$\begin{aligned} 4K &= |(\overline{1-z})(\omega^2-\omega z) - (\overline{\omega^2-\omega z})(1-z)| \\ &= |(1-\overline{z})(\omega^2-\omega z) - (\omega-\omega^2\overline{z})(1-z)| \\ &= |\omega^2 - \omega z - \overline{z}\omega^2 + \overline{z}\omega z - (\omega - \omega z - \omega^2\overline{z} + \omega^2\overline{z}z)| \\ &= |\omega^2 - \omega + (\omega-\omega^2)\overline{z}z| \end{aligned}$$

But since $\omega = (-1+i\sqrt{3})/2$ and $\omega^2 = (-1-i\sqrt{3})/2$, we have $\omega-\omega^2 = i\sqrt{3}$. Further $|z|^2 = z\overline{z}$. Thus,

$$4K = |\sqrt{3}i|z|^2 - i\sqrt{3}| = \sqrt{3}(1-|z|^2)$$

This shows that K depends only on $OP = |z|$.