Assignment 3.

Given October 11, due October 25.

Objective: A more practical example of PDE based pricing.

We want to price an American style call option on a stock with the following extra feature: the holder may at any time put a barrier into the call and receive payment M. Without the barrier, the option is an American call struck at K. With the barrier, the option becomes worthless if the underlying asset value at any time (after the barrier is inserted) exceeds L. At any time, the holder may convert from barrier free to barrier and receive payment M at that time. The time to expiration is T and the initial spot price is S_0.

1. Suppose the volatility is $\sigma(S)$, explain the hedging argument that leads to a PDE (or more than one) for the price. Explain how the various early exercise decisions are implemented in the PDE.

2. Suppose that $\sigma = \sigma_0$ is a constant. Find the option value if $S_0 = 100$, $K = 120$, $L = 140$, $\sigma_0 = .1$, $T = 4$, and $r = .06$, $M = 10$.

3. Find the sensitivities with respect to S_0 and σ_0 using each of two methods: finite differences with the code from part 2 and by solving the PDE satisfied by the sensitivity. You find the PDE satisfied by the sensitivity by differentiating the original PDE with respect to the parameter. This new PDE will have more terms than the original one. Approximate all the terms by finite differences. The hard part is getting boundary conditions for the sensitivities. You might be able to guess them by looking at numerical computations or by computing the sensitivities via finite differences.

4. Repeat part 2 with a skewed volatility

$$\sigma S = \sigma_0 + \sigma_1 (S - S_0)$$

with $\sigma_1 = .0012$.