Scientific Computing, Courant Institute, Fall 2018
http://www.math.nyu.edu/faculty/goodman/teaching/ScientificComputing2018/ScientificComputing.html
before doing any work on the assignment.

Final exam practice

Information

- The final exam is Thursday, December 20 in room 1302 from 5:10 to 7pm.
- The exam starts promptly at 5:10, don't be late.
- You are allowed one standard size ($8\frac{1}{2}'' \times 11''$) sheet of paper with any information you like. No other information or electronics are allowed.
- Write all answers in one or more blue books provided. Hand in only the blue books.
- Write your name on each blue book and number them (e.g. 1 of 1, 2 of 3 etc.)
- You will receive 20% credit for question if you write nothing.
- Anything you do write may be counted against you if it is wrong.
- Cross out anything you think is wrong. If you have two answers, the wrong one will count against the right one.
- On multiple choice or true/false questions, give a few words or sentences of explanation. You may lose points even with a correct answer, if it isn't explained.
- Suppose that W_t is standard Brownian motion and $dX_t = W_t^2 dt$. Evaluate the quadratic variation

Practice questions

True/False

1. The local convergence for gradient descent optimization is faster than for Newton's method.
2. Spline interpolation applies when the interpolation points are not evenly spaced.
3. FFT based trigonometric interpolation applies when the points are not evenly spaced.

Multiple choice

1. Which of the following numbers is represented exactly in the IEEE double precision floating point standard?

   (a) 1.33

   (b) e

   (c) 1.25×10^{20}

   (d) $1 + 1.23456789 \times 10^{-13}$.

2. Suppose A and B are $n \times n$ matrices and we want to calculate the matrix product $C = AB$. This can be done in vector Python with a loop over i and j containing `C[i,j] = np.dot(A[i,:],B[:,j])`. Or it can be done with the single instruction `C=np.matmul(A,B)`. Which of the following is true about the relative speed for large n?

   (a) The two methods are about as fast.

   (b) The `np.matmul` method is faster because Python is interpreted, not compiled.

   (c) The `np.matmul` method is faster because it used block matrix multiplication that makes better use of cache memory.

   (d) The `np.dot` method is faster because dot products are faster than matrix multiplies.

3. Which of the following operations is not computed to high relative accuracy in double precision floating point using built-in Python procedures for special functions:

   (a) $\sqrt{123456789} + \sqrt{123456788}$ (the numbers differ in the last digit)

   (b) $\sqrt{123456789} - \sqrt{123456788}$ (the numbers differ in the last digit)

   (c) e^{1000}

   (d) e^{-1000}

Full answer questions

1. Find an estimator of the form

$$f'(x) = \frac{af(x) + bf(x+h) + cf(x+2h)}{h} + O(h^p) \ .$$

   Find the coefficients a, b, and c and the optimal order, p.

2. Find an estimator of the form

$$\int_0^h f(x)dx = h\left[af(0) + bf(h) + cf(2h)\right] + O(h^p) \ .$$

   Find the coefficients and the best possible p.

3. Suppose $f_0 = f_1 = g_0 = g_1$ and

$$f_{n+1} = f_n + f_{n-1}$$
$$g_{n+1} = \frac{3}{2} g_n + \frac{1}{3} g_{n-1} \ .$$

Show that $f_n / g_n \to 0$ as $n \to \infty$ exponentially. Hint: (to the accuracy given) $\sqrt{5} = 2.236$ and $\sqrt{2.25 + \frac{4}{3}} = 1.893$.

4. The solution to a differential equation is calculated with time step Δt. The exact answer is 4.28×10^7. The computed errors at time T are

Δt	.1	.05	.025
error	32.61	4.17	.522

Estimate the order of accuracy of the method from this information.