Sample questions and instructions for the quiz
Thursday, March 21, whole period

Corrections: (none yet)

Instructions.

- You may not any materials or electronics except ...
- You may bring one sheet of paper (a cheat sheet) with anything written on it. The only requirement is that you prepare it yourself and that you can read it without assistance.
- You get 20% of the points on any question if you leave it blank.
- Anything you write that is wrong will count against you, even if you also write the correct answer. If you change your mind about an answer, cross out the part you think is wrong.
- For True/False and multiple choice questions, write a few words or a few sentences explaining your answer.
- There are more sample questions here than there will be on the actual quiz

Material on the midterm exam: Everything we've done in class until the Tuesday before the exam. This includes material from the first part of the course that was covered in the quiz. The quiz and practice quiz are a source of practice questions for that material. These questions are on material that was not covered in the quiz.

Difficulty of the practice questions: Some of these practice questions are more complicated than would be possible on the actual exam, but they are good practice for actual exam questions. As for the quiz, the actual midterm exam will be shorter than this so that you can do it in one class period. The real exam will not have questions on every topic here (or the quiz).

True/False. For each statement, say whether it is true or false and explain.

1. According to the Black Scholes model, the expected rate of return of every asset is the same, regardless of the volatility, in the risk neutral measure.
2. The Black Scholes model determines the volatility of an asset in the risk neutral measure.

3. The Δ of a put option is always negative, regardless of the style (European or American) or the other parameters.

4. The Γ of a put option is always positive, regardless of the style (European or American) or the other parameters. (It is possible to prove the answer to this question, but it's enough here to remember the shape of the graph of the price as a function of s.)

Multiple choice

1. Suppose X is a random (positive or negative) cash flow with probability density $p(x)$. In finance, it may be important to know whether the density has fat tails because

   (a) The price of a fat tailed distribution is higher than the price of a thin tailed distribution.

   (b) The risk of a large loss (many standard deviations from the mean) is relatively high compared to a thin tailed distribution.

   (c) Fat tailed distributions are very rare in real financial situations.

   (d) If the model predicts a fat tailed distribution, then the model is wrong.

2. Which of the following is *not* a parameter in the Black Scholes option pricing formula for European style options?

   (a) r (risk free rate)

   (b) σ (asset price volatility)

   (c) K (option strike price)

   (d) Δt (time step for CRR binomial tree)

3. Which of the following is a correct fact about put-call parity?

   (a) It is a relation between put and call prices for European style options with the same strike price and expiration time.

   (b) It is a relation between put and call prices for American style options with the same strike price and expiration time.

   (c) It is a relation between put prices for two European style options with the same strike prices but expiration times.

   (d) It is true even if the binomial tree or Black Scholes model is wrong.

General questions

1. Suppose the owner of a share of stock at time t_* recieives a *dividend*, which is a payment of D. Use an arbitrage argument (called "buying a dividend") to find a formula for the jump in the stock price at time t_*. The jump is

$$J = \lim_{\epsilon \to 0, \epsilon > 0} S_{t_*+\epsilon} - S_{t_*-\epsilon} \ .$$

   Assume that the stock price is continuous except at t_*.

2. Order the following distributions by the size of their "upside" tails, the tails as $s \to \infty$. Go from the thinnest tail to the fattest tail.

   (a) A Gaussian, $p(s) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(s-\mu)^2}{2\sigma^2}}$

   (b) A lognormal, $S = e^X$, where X is normal with some mean and variance.

   (c) A *Student T* random variable with probability density $p(s) = C \frac{1}{(1+a(s-\mu)^2)^n}$

   (d) S is uniformly distributed in the interval $[a, b]$.

3. Consider a two step CRR binomial tree model, where the final values are uuS_0, udS_0 and ddS_0. Suppose an option pays \$1 for uuS_0 and zero for the other two outcomes. Take $u = 1.2$ (for a 20% increase) and $d = .8$. Takek $r = 0$.

   (a) Calculate the value of the option in the two intermediate states after the first step and then the value of the option at the beginning.

   (b) Calculate the Deltas (number of shares of stock in the replicating portfolio) at the two intermediate states and at the beginning.

   (c) If the stock goes from S_0 to uS_0 in the first step, how many shares of the stock would be bought (sold, if the number is negative)?

4. Let $N(x)$ be the cumulative normal distribution function used in the Black Scholes formula. Show that

$$N(d) = 1 - N(-d) \; . \qquad (1)$$

   Hint: If Z is a standard normal random variable, then $-Z$ is also a standard normal (explain). If $Z < d$ then $-Z > -d$.

5. Use the Black Scholes formulas for C and P (call and put prices with the same T and K and r and σ) to show that they satisfy put-call parity. The formula (1) will come in.

6. An option is *deep in the money* if the price of the underlier is so far from the strike price (below the strike for a put, above for a call) that it is nearly certain that the option will be exercised. A put or call option that is guaranteed to be exercised (by contract) isn't an option, but a forward contract. Show that the Black Scholes formula is approximately equal to the forward price for a deep in the money call. Hint: use the facts $N(d) \to 0$ as $d \to -\infty$ and $N(d) \to 1$ as $d \to \infty$.

7. A *power law* with exponent β is a function $f(s) = As^\beta$, where A is a constant. Consider the Black Scholes theory for a European style option with a power law payout at the expiration time T. Let $f(s,t)$ be the Black Scholes theoretical price, when $t \le T$. Show that f is a power law function of s for each t, with the same exponent β. That is, show that

$$f(s,t) = A(t)s^\beta \; . \qquad (2)$$

Hint: Plug this *ansatz* (2) into the Black Scholes partial differential equation (the "Black Scholes equation") and show that the equation is satisfied if $A(t)$ satisfies the right differential equation.

8. Write an `R` function `IM(S, K, n)` that returns the number of data values in the array S larger than the number `K`. If S_i for $i = 1, \ldots, n$ is a collection of numbers and K is given. We want

$$M = \# \{ i \text{ with } S_i > K \} \ .$$

The function `IM(S,K,n)` (for "In the Money") should calculate and return `M`.