Sample questions and instructions for the final exam
Thursday, May 16, 6:00 - 7:50 pm.

Come 5 minutes early so the exam can be distributed by 6.

Corrections: (none yet)

Instructions.

- You may not any materials or electronics except ...
- You may bring one sheet of paper (a cheat sheet) with anything written on it. The only requirement is that you prepare it yourself and that you can read it without assistance.
- You get 20% of the points on any question if you leave it blank.
- Anything you write that is wrong will count against you, even if you also write the correct answer. If you change your mind about an answer, cross out the part you think is wrong.
- For True/False and multiple choice questions, write a few words or a few sentences explaining your answer.
- There are more sample questions here than there will be on the actual quiz

Material on the final exam: Everything we've done in class. Please use the sample quiz and sample midterm for sample questions on material covered in the first half of the course.

Difficulty of the practice questions: Some of these practice questions are more complicated than would be possible on the actual exam, but they are good practice for actual exam questions.

True/False. For each statement, say whether it is true or false and explain.

1. If X is a square matrix, then the singular values of X^2 are squares of singular values of X.
2. If C is a square matrix, then the eigenvalues of C^2 are squares of the eigenvalues of C.
3. Suppose $w_1, \ldots, w_n$ is a set of portfolio weights for assets $X_1, \ldots, X_n$. The expected return is $\mu_R = w^t\mu$ and the variance is $\sigma^2 = w^tCw$. Suppose that there is no portfolio with the same return with a smaller variance. Then there is no portfolio with variance σ^2 and larger expected return.

4. If $f(x)$ and $g(x)$ are differentiable functions with $g(x_*) = a$, and if $\nabla f(x_*) = \lambda \nabla g(x_*)$, then x_* maximizes f subject to the constraint $g(x) = a$.

5. If C is an $n \times n$ symmetric matrix with entries $\sigma_{jk} \geq 0$ for all j and k. Then $w^t C w \geq 0$ for all vectors $w \in \mathbb{R}^n$. Hint: try $C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

6. If S is a lognormal random variable, then $\mathrm{var}(S) < \infty$.

7. If X is a Gaussian random variable then $\mathrm{E}\left[e^X\right] < \infty$ and $\mathrm{E}\left[e^{X^2}\right] < \infty$.

8. Suppose Z_1 and Z_2 are uncorrelated random variables. Then Z_1^2 is uncorrelated with Z_2^2.

Multiple choice

1. Which of the following is a consequence of the Cauchy Schwarz inequality?
   (a) If $\mathrm{E}\left[X^2\right] = 4$ then $|X| \leq 2$.
   (b) If $\mathrm{E}[X] = 2$ then $\mathrm{E}\left[X^2\right] \leq 4$.
   (c) If $p(x)$ is the probability density of X, then $\mathrm{E}[X] = \int_{-\infty}^{\infty} x p(x)\, dx$.
   (d) If $\mathrm{E}\left[X^2\right] = 4$ then $-2 \leq \mathrm{E}[X] \leq 2$.

2. Suppose $X_{t,k}$ is the price of asset k at time t. Suppose X is the data matrix formed from these numbers. The singular value decomposition is $X = U \Sigma V^t$. Where would you find a time series most correlated with the fluctuations in the price of asset k?
   (a) The first column of U.
   (b) The first row of U.
   (c) The first column of V.
   (d) The diagonal entries of Σ.

3. Why does default correlation create greater risk for a diverse portfolio of bonds?
   (a) Default correlation increases the risk that a bond will default.
   (b) Correlation is an important factor in bond ratings.
   (c) Correlations larger than one violate the Cauchy Schwarz act.
   (d) Correlation increases the probability that the portfolio payments will be significantly lower than the expected payments.

4. What is determined by bond default intensity?
   (a) The effect that a bond default would have on its coupon payments.
   (b) The probability that a bond that hasn't defaulted will default in the next short time period.

(c) Correlations between defaults of different bonds.

(d) The risk that interest rates will fluctuate.

5. Many corporate bonds are *callable*. The issuing company has the right to refund the principal and end coupon payments at any time? What would this right be called in finance, from the company's point of view?

   (a) An American style put option for the bond.

   (b) A European style call option on the bond.

   (c) An American style call option on the bond.

   (d) A risk free arbitrage opportunity.

General questions

1. Suppose C is an $n \times n$ symmetric matrix. Find a formula for $\nabla \left(w^t C w\right)^4$. Hint: use the chain rule.

2. Suppose Z_1 and Z_2 are uncorrelated Gaussian random variables. Suppose $X = aZ_1 + bZ_2$ and $Y = cZ_1$. Calculate the correlation coefficient between X and Y. Explain the fact that the correlation is independent of c but depends both on a and b. (Hint for the last part: consider the problem of predicting X using Y.)

3. Suppose an `R` script has $n \times n$ matrices `A` and `B` so that the mathematical matrices satisfy $AB = I$. Suppose `u` and `f` are n component vectors. Write the `R` code to find x with $(A + uu^t)\, x = f$ using the Sherman Morrison formula. (Hint: put the Sherman Morrison formula on your cheat sheet along with Black Scholes.) Use only matrix and vector multiplication involving u, f, and B.

4. Suppose the `R` function `randp()` returns a random variable with probability density $p(x)$. Suppose each call to `randp()` returns an independent random variable with the same density.

   (a) Write an `R` script that uses N independent samples (found using N calls to `randp()` to estimate $\Pr(X > a)$. Assume that `a` has been defined.

   (b) Write a script that uses the same samples to estimate $\mathrm{E}[X]$.