

Ordinary Differential Equations
Homework 10

Given: December 1

Due: December 6

1. Consider the function $f(x, y) = \begin{pmatrix} x \cos(y) \\ x \sin(y) \end{pmatrix}$. Find $\begin{pmatrix} x^* \\ y^* \end{pmatrix}$ so that $f(x^*, y^*) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Calculate the Taylor approximation to f about $\begin{pmatrix} x^* \\ y^* \end{pmatrix}$ up to and including quadratic terms:

$$f(x^* + u) \approx f(x^*) + Au + Q(u, u). \quad (1)$$

Here we have used x^* to represent the two component vector $\begin{pmatrix} x^* \\ y^* \end{pmatrix}$. There are four numbers in A and six in Q . Estimate $\Delta f = f(x^* + u) - f(x^*)$ using the linear part $\Delta f \approx Au$, and the full quadratic approximation (1), when $u_x = .2$ and $u_y = .3$. Calculate Δf exactly (using a calculator). Comment on the accuracy of the linear and quadratic approximations.

2. Section 9.1, # 13.
3. Section 9.2, # 4.
4. Section 9.3, # 1, 3 (Ignore the stuff about being “almost linear”. Any differentiable function has this property. It is the very definition of differentiability), 5, 9, 12 (These three are time consuming but it’s worth the time. This is one of the high points of the course, after chapter 7.)
5. Section 9.4, # 1 (Do part b first and part a last. We draw in the direction field and some trajectories after we’re done with the lamppost calculations.).
6. Section 9.5, # 2 (Do part b first and part a last. We draw in the direction field and some trajectories after we’re done with the lamppost calculations.).