Ordinary Differential Equations

Homework 10

Given: November 18
Due: November 22

1. For an $n \times n$ matrix A, the *transfer function*, also called *resolvent*, is $R(s) = (sI - A)^{-1}$. Show that $R(s)$ is defined for all complex numbers that are not eigenvalues of A.

2. Show that we may find a particular solution of $\dot{x} = Ax + ve^{st}$ of the form $x(t) = we^{st}$ using the resolvent provided s is not an eigenvalue of A.

3. Compute $R(2)$ for the matrix

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ 1 & 2 & 2 \end{pmatrix} .$$

   Use it to find a particular solution to the equation $\dot{x} = Ax + ve^{2t}$ where $v = (1, 0, -1)^t$.

4. For a 2×2 matrix

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} ,$$

   verify the formula

$$M^{-1} = \frac{1}{\det(M)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

   by checking that the product of the two matrices is I.

5. Use the formula from question 4 to calculate $R(s)$ where

$$A = \begin{pmatrix} -1 & 1 \\ -4 & 0 \end{pmatrix} .$$

6. Use your answer to question 5, and of course complex exponentials, to find a particular solution to

$$\dot{x} = Ax + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cos(t) .$$

7. Section 7.6, # 4, 5 (Take real or imaginary parts of complex solutions.), 9, 26.

8. Section 7.7, # 3, 11.

9. Section 7.5, # 1, 11, 13.