

Derivative Securities, Courant Institute, Fall 2010

<http://www.math.nyu.edu/faculty/goodman/teaching/DerivSec10/index.html>

Always check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on Derivative Securities) before doing any work on the assignment.

Practice for the final exam

Corrections: (none yet)

Important:

- The final exam is on Wednesday, December 22, from 7:10 to 9 pm in the usual room, 109.
- You only may bring one piece of paper with whatever information you want to put on it. You must be able to read this without magnifying lenses or electronics. No other materials, electronics, or books are allowed.
- Cross out anything you think is wrong. You may have points deducted for incorrect statements even if you also give the correct answer.
- You will receive partial credit for declining to answer a question or leaving it blank. You will lose this partial credit if you give a wrong answer.
- You may not use and will not need a calculator.
- Write all answers in the blue exam books. If you use more than one book, put your name on each book and label the books, e.g. **book 2 of 3**.
- The questions below are examples of the kind of questions that will be on the actual final. The questions on the final will be different, though some may be similar.
- There is a review session on Thursday, Dec 16, from 7 to 9pm in room 101.
- The actual exam will be much shorter than this.
- Also look at the sample from last year. Avoided duplication but many of the questions there are relevant this year too.

The Questions

Part 1, True/False. *In each case, indicate whether the statement is true or false. Please explain your answer in a few words. You may receive no credit for a correct answer with no explanation.*

1. If the futures price of an asset is not equal to the forward price $S_0/B(0, T)$ (S_0 being current spot price and $B(0, T)$ the current risk zero bond rate for bonds maturing on the delivery date), then there is an arbitrage opportunity.
2. If the Δ of a stock option you own is positive and the Γ is zero, then you make money if the underlying stock goes down a little.
3. If the Γ of a stock option you own is positive and the Δ is zero, then you make money if the underlying stock goes down a little.
4. In the Black Scholes theory, the Γ of an option increases as the option approaches its expiration date.
5. The Black Scholes implied volatility of an option is constant over the life of the option in real markets.
6. In principle it is impossible to buy an American style option in the early exercise region because all of them would be exercised already.
7. If you own a number of put options that are near the money, sell them and use all the funds to buy deep out of the money puts, your Gamma decreases because Gamma is largest near the money.
8. The historical default rate of bonds of a given credit rating is a reasonable estimator of the current credit spread for such bonds.
9. The rate in a forward rate agreement to buy a bond at time t that matures at time $T > t$ is known at the time the agreement is agreed to.
10. If two bonds are identical except that one is callable, then the callable bond is worth more than the one that is not callable.
11. A parallel shift of the yield curve effects the price of long dated bonds more than it effects the price of short dated bonds.
12. The five LIBOR zero rate (the yield curve for LIBOR) is determined by the prices of five year LIBOR bonds.
13. It would be possible to replicate the income stream of a coupon bearing bond using a portfolio of zero coupon bonds if such zero coupon bonds were available in the marketplace.
14. The dividend rate of a stock is irrelevant in determining its forward price.
15. The Black Scholes formula is not useful when trading exchange traded options because the prices of these options are already known in the exchange.
16. Bermudian style options have an early exercise feature.

17. Monte Carlo is faster and more accurate as a way to compute risk neutral prices than the Euler method.

Part 2, multiple choice. *In each case, indicate which of the choices is correct. Please explain your answer in a few words. You may receive no credit for a correct answer with no explanation.*

1. For most exchange traded options on exchange traded equities, the implied Black Scholes volatility of a put option is
 - (a) Independent of the strike and expiration date as the Black Scholes theory says it should be
 - (b) Largest for deep out of the money options
 - (c) Largest for options near or at the money
 - (d) Impossible to determine from market data alone
2. The early exercise feature of American style options refers to
 - (a) The right of the option holder to exercise the option at any time before it expires
 - (b) The right of the option writer to cancel the option
 - (c) The fact that American style markets close before European style markets close
 - (d) The fact that American traders go to the gym before markets open
3. In discussing interest rates, one *basis point* refers to
 - (a) One percent of one percent
 - (b) Ten percent of one percent
 - (c) One percent
 - (d) A bullet point in a presentation slide
4. The term *credit spread* refers to
 - (a) The range of possible yields on bonds offered by different companies
 - (b) The difference between the yield on a one year and a five year Treasury
 - (c) The difference in yield of a bond that depends on the estimated likelihood of the bond issuer to default.
 - (d) A fancy meal paid for with a Visa card.
5. The term *risk neutral measure* refers to
 - (a) The extent to which investors are risk neutral
 - (b) The fact that large financial institutions have portfolios so diversified that they do not care about risk

- (c) A probability distribution used to predict prices of options
 - (d) The probability distribution of the values of options when they expire
6. In a general discrete one period model, if there is a set of probabilities so that the price of each asset is equal to its expected value in the probabilities, then
- (a) The model contains an arbitrage opportunity
 - (b) The model is complete and arbitrage free
 - (c) The model is complete but not necessarily arbitrage free
 - (d) The model is arbitrage free but not necessarily complete
7. An option that allows the holder to choose the best price over a given period is called a
- (a) Asian
 - (b) Azoran
 - (c) Bermudan
 - (d) Lookback
8. Consider a put option on an asset S . Suppose an option is purchased on an asset that had the following most recent daily closing prices: 9.7, 9.8, 9.9, 10. After that, the next daily closing prices were 11.1, 9.2, 11.3, 9.4, 10.5. Which of the following Greeks is most relevant to explaining the return on the option at the end of those five days:
- (a) Delta
 - (b) Gamma
 - (c) Vega
 - (d) Rho
9. The coefficients in the Euler/trinomial tree method for valuing American style options are determined by
- (a) Mathematical calculations using Taylor series and the Black Scholes equation
 - (b) A discrete hedging argument like the one used for the binomial tree
 - (c) The requirement that the tree be recombining
 - (d) The early exercise price.
10. The Euler/trinomial tree method for valuing American style options has certain advantages over the binomial tree method. Which of these are advantages
- (a) The trinomial tree may be independent of the parameters in the model

- (b) The binomial tree cannot be used for American style options
 - (c) The trinomial tree is recombining even if the volatility is not constant
 - (d) All of the above
 - (e) (a) and (c) above
11. The copula is a way to model
- (a) The expected return of a single asset
 - (b) The risk of a single asset
 - (c) The expected return of a bundle of a large number of assets
 - (d) The risk of a bundle of a large number of assets
12. Which of the options portfolios has a payout most resembling a δ -function?
- (a) A butterfly
 - (b) A bull spread
 - (c) A bear spread
 - (d) A straddle

Part 3, Full answer

1. Be prepared to compute an option price in a binary model.
2. Suppose $dS_t = rS_t dt + \sigma S_t dW_t$. Give a formula for $\Pr[S_T < S_0]$.
3. Suppose $dr_t = \sigma\sqrt{r_t}dW$, and $X_t = \sqrt{r_t}$. Show that $dX_t = \frac{\sigma}{2}dW + a(X_t)dt$. Find a formula for $a(x)$.
4. Suppose that the default intensity of a zero coupon bond is $\lambda = 3\%/\text{yr}$ and that the market price of this bond is such that its credit spread above LIBOR is 2%. What is the implied recovery rate implicit in this yield spread?
5. Consider the PDE $\partial_t f + \frac{1}{2}\partial_x^2 f + x^2 f = 0$. Suppose $f(x, T) = 1$. Give an exponential quadratic ansatz that gives a formula for the solution at earlier times. Write the differential equations one would have to solve to find the parameters in the ansatz.
6. More to come – have to post now.