Derivative Securities,

Derivative Securities, Courant Institute, Fall 2008

http://www.math.nyu.edu/faculty/goodman/teaching/DerivSec08/index.html

Practice for the Final Exam

About the final exam

1. The final exam is on Wednesday, December 17, from 5:10 to 7pm or 7:10 to 9 pm in the usual classroom, depending on your section.

2. The exam will be closed book and closed notes. You will not need nor be allowed a calculator. You will be allowed one $8\frac{1}{2} \times 11$ piece of paper, a *cheat sheet*, with anything you like written on it. You may not use a magnifying glass to read it.

3. In the true/false section, first indicate your answer (true/false), then give a few words explaining the answer. A correct T/F without a reason will not get any points. In the multiple choice section, explain your answer with a few words.

4. You may have points deducted for anything you write that is incorrect even if you also give the correct answer. Cross off anything you feel is incorrect.

5. You will receive 20% of the points for any question if you leave it blank or state that you are unable to answer it. You may have points subtracted for anything incorrect you write. If you have no idea how to answer a question, you will get more points by saying so than by giving a guess that is unlikely to be correct.

Part 1, True/false

1. An option on a stock has a payout $V(S(T))$. The Black Scholes PDE allows you to find the price of such an option for very general payouts $V(s)$, not just for vanilla puts and calls.

2. The Black Scholes PDE allows you to specify a present price structure $f(s_0, 0)$ and design a payout function $V(s_T)$ to achieves this price structure.

3. Two bonds have the same principal and coupons, payment dates, and maturity date. They sell at different discounts. The one that sells at a higher discount (sells for less) has a shorter duration.

Part 2, multiple choice

1. I have a formula for $Y(t)$, the effective interest rate for money borrowed now and repaid at time t. This formula is called the (see Hull, Chapter 28)
   (a) Volatility of interest rates (b) Term structure of interest rates
   (c) Option price of interest rates (d) Hedge ratio of interest rates

2. If $f(t)/g(t)$ is a martingale, which of the following is constant, in the sense of being independent of t?
   (a) The expected return on $f(t)$ (b) The market price of risk
   (c) The expected value of $f(t)/g(t)$ (d) The volatility

3. We use the risk neutral measure for pricing stock options because

   (a) Large institutions have so much capital that they are insensitive to risk

   (b) It is easier to estimate the risk neutral measure than the historical measure from historical price data

   (c) Risk neutral prices are generally lower, so we can enter options contracts at less cost

   (d) Arbitrage pricing theory suggests that future option prices are determined by the risk neutral measure.

4. Which data would we be most likely to use to construct a yield curve?
   (a) NASDAQ (b) the London EDX
   (c) FEMA (d) LIBOR

5. A loan is being negotiated under which the counterparty will pay the one month (answer to question (4) rate each month for five years and then repay the principle. One of the parties asks that if that rate is more than 7% (annualized), then he should pay 7% instead. This extra aspect of the loan contract is called
   (a) a cap (b) a limit order
   (c) a swap (d) a forward contract

6. The implied volatility implicit in the near the money puts and calls suggests that a stock should move about 5% in the next month. I believe it will move much less. The price is S. To profit from that belief (if true) I should (draw the payout diagrams and name them)

   (a) buy one share of stock and short Δ shares of the at the money option

   (b) buy one put with strike $K_1 < S$ and one call with strike $K_2 > S$

   (c) buy a call struck at $K_1 = S - \widetilde{S}$, (with $\tilde{S}$ small but positive), buy a put struck at $K_2 = S + \widetilde{S}$, and sell a call and a put both struck at S.

   (d) buy a call struck at $K_1 < S$ and sell a put struck at $K_2 > S$.

7. The derivative on an option price with respect to the volatility parameter is traditionally called
   (a) Delta (b) Alpha
   (c) Vega (d) Gamma

8. The *smooth pasting* condition for American style options is the fact that
   (a) The Gamma of the option is a smooth function of the strike price
   (b) The early exercise boundary is a smooth function of time
   (c) The Delta of the option is ± 1 (call or put) at the early exercise point
   (d) The early exercise boundary converges to the strike price as time approaches the expiration time.

9. The Gamma of a vanilla option (European or American style) is greatest
   (a) near the strike price, close to expiration
   (b) deep out of the money, close to expiration
   (c) near the strike price, far from expiration
   (d) near the early exercise boundary

Part 3, long answer

1. A certain country issues risk free treasury bounds that pay constant coupon C once per year. In the currency of that country, P_k is the market price of a treasury that matures in k years. Describe a way to compute the risk free yield curve in that country from these numbers.

2. People are betting on whether Kitty Forest will win an upcoming golf tournament. The betting is such that you can pay \$40 for a payment of \$200 should she win. What is the risk neutral probability that she will win.

3. Suppose in the problem above there are two tournaments. You can buy a \$200 payout (should she win) on either individual tournament for \$40. For \$10 you can buy a \$200 if she wins both (you get nothing if she does not win both). What is the implied correlation between the two tournaments? That is, $X_1 = 1$ if she wins tournament 1 and $X_1 = 0$ otherwise, $X_2 = 1$ if she wins the second tournament and $X_2 = 0$ otherwise. We want the implied correlation between X_1 and X_2.

4. In a one period binomial model, suppose the forward price today is $\mathcal{F}_0 = 20$ and that the forward price at time T will have one of the two values

$$\mathcal{F}_T = \mathcal{F}_0 \pm \delta\mathcal{F}$$

   Suppose the zero rate is $B(0,T) = .8$ and the market value of an call option with strike $K = 20$ is 4. Find the implied value of $\delta\mathcal{F}$. (This involves more arithmetic than might be practical on the real exam.)

5. Write a formula for the Black Scholes price of a put, assuming that the present price is 100, the strike price is 110, the risk free rate is so that $e^{rt} = 1.1$ with $t = 1$ (slightly less than 10%). The option expires one year from now. The volatility is 20%/year. You may express the answer in terms of the N function (cumulative normal), but give numerical values to any arguments to the N functions you use.

6. In an interest rate swap, we trade a floating rate payment for a fixed rate. If the notional is L, which leg of this swap has a simple known value? What is it?
7. Use Black's model to give the value of an interest rate cap (numbers supplied on the exam).