Fall 2004 Calculus I, sections 4, 5, 6, Courant Institute of Mathematical Sciences, NYU.

Homework 4, due October 4

Self check (not to hand in, answers are in the back of the book):

Section 1.6: 17, 25, 29, 61, 69.

Section 2.5: 1.

Section 3.6: 5, 17, 21, 27, 33, 43.

To hand in:

Section 1.6: 18, 24, 30, 62, 70.

Section 2.5: 2, 36 (hint: multiply by $\frac{bx}{ax}$).

Section 3.6: 4, 18, 22, 32, 44.

More problems (to hand in)

1. For small x we have the approximations $\sin(x) \approx x$ and $\cos(x) \approx 1 - x^2/2$. For any y and x we have the angle sum formula

$$\sin(y + x) = \sin(y)\cos(x) + \cos(y)\sin(x) \; .$$

Using these, we can get approximations to $\sin(y + x)$ if we know $\sin(y)$ and $\cos(y)$, and x is small. Use this approximation to estimate $A = \sin(50^o)$ and $B = \sin(60^o)$ using $y = 45^o$, $\sin(45^o) = \cos(45^o) = 1/\sqrt{2} \approx .70711$. Call the approximate values a and b respectively. Use a calculator to evaluate $A - a$ and $B - b$. Which of A or B is approximated more accurately? Why? (Be careful to convert to radians as needed.)

2. An airplane is flying at a constant height of 3000 Meters (just under $10,000$ feet) directly toward me at 500 Km/h (approximately 300 $Miles/h$). Its horizontal distance from me is 6 Km, so that its total distance is just under 7 Km.

a. What angle does the line from me to the plane make to the horizontal?

b. How fast is that angle increasing? Give the answer in radians per hour, then convert to radians per minute or radians per second, whichever is the most informative. (Hint: let $x(t)$ be the horizontal distance, $\theta(t)$ the angle, and H the (constant) height. Then $\tan(\theta) = H/x$. Differentiate both sides with respect to t (the derivative of the left side must equal the derivative of the right side). Use the chain rule for both sides. On the left we get the unknown $d\theta/dt$ and on the right the known dx/dt.)