Fall 2004 Calculus I, sections 4, 5, 6, Courant Institute of Mathematical Sciences, NYU.

Homework 1, due September 13

Self check (not to hand in, answers are in the back of the book):

Section 2.1: 1, 17, 27, 55 (use a calculator).

Section 2.2: 5, 7, 21, 29 (do it by hand).

Section 2.3: 1, 11, 15, 39, 57.

To hand in:

Section 2.1: 6, 56 (use a calculator).

Section 2.2: 2, 10.

Section 2.3: 2, 14, 20.

More problems (to hand in)

1. Let $A(n)$ be the number of pairs of *integers* (i, j) with $1 \le i \le n$ and $1 \le j \le n$. Let $B(n)$ be the number of pairs with $i < j$.

a. Show that $A(n) = n^2$. Don't worry if this seems too easy, it is.

b. Show that $B(n) = n(n-1)/2$. It is OK to verify this by counting dots in a picture for $n = 2, 3, 4$.

c. Find $\lim_{n\to\infty} B(n)/A(n)$ using algebra and the fact that $\lim_{n\to\infty} 1/n = 0$.

d. Draw a picture for somewhat large n (say $n = 10$) to illustrate the answer to part c.

2. The *floor* or *integer part* of x, written $\lfloor x \rfloor$ is the largest integer not greater than x. For example, $\lfloor 4.3 \rfloor = 4$, $\lfloor 5 \rfloor = 5$, and $\lfloor -3.4 \rfloor = -4$.

a. For which values of x is $f(x) = \lfloor x \rfloor$ continuous?

b. For which values of x do left and/or right limits exist?

c. Carefully draw the graph of $f(x)$ for x in the range $1 < x < 5$ using open and closed circles at points of discontinuity as done in SHE.

d. We say that $\lim_{x\to\infty} g(x) = L$ if, for any $\epsilon > 0$ there is an $R > 0$ so that if $x > R$, then $|g(x) - L| \le \epsilon$. Find the value of L for $g(x) = \lfloor x \rfloor / x$, and show that $R = 1/\epsilon$ (or possibly $R = 1 + 1/\epsilon$) works.