USING PRINCIPAL COMPONENT ANALYSIS TO EXPLAIN TERM STRUCTURE MOVEMENTS: PERFORMANCE AND STABILITY

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Author: Gloria M. Soto

Affiliation: Departamento de Economía Aplicada, Universidad de Murcia
Address: Campus de Espinardo, 30100 Murcia, Spain
E-mail: gsoto@um.es

Abstract:

This paper evaluates the performance of a kind of interest rate model that has increasingly been attracting the attention of the financial industry in recent years and which relies on principal component analysis to extract risk factors. Focusing on the Spanish bond market, our empirical analysis reveals that interest rate movements can be summarized by three principal components, related to the level, the steepness and the curvature of the yield curve. This three-principal component model is able to offer a balanced explanation of interest rate shocks and bond returns across maturities and overcomes typical one- and two-factor interest rate models. However, our results also reveal some variations with time in the principal components that point to the need to recognize the dynamic volatility structure of interest rates when dealing with factor models.

JEL classification: E43; G12

Key words: interest rate, bond, risk management, factor model, principal component

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1. Introduction

Principal component analysis (PCA) is a powerful statistical tool used in a variety of real-life systems. Its key assumption is that the multitude of factors that affect a system can be neatly summarized by a few uncorrelated composite variables, called principal components, which provide a parsimonious description of the system's dynamics. This reduction in dimensionality is particularly useful in finance, since asset prices are affected by thousands of economic variables that are difficult to translate into a rigorous price model.

Although PCA was first applied to equity markets in the financial area, both academics and professionals have extended this technique to fixed income markets in recent years in an attempt to offer a new framework for interest rate risk management. The works of Barber and Copper (1996), Bliss (1997), Falkenstein and Hanweck (1997), Singh (1997), Wadhwa (1999) and Golub and Tilman (2000), among others, are examples of the current growing interest in PCA for interest rate risk analysis.

From a practical perspective, the use of PCA in bond markets has revealed that only three principal components (related to the level, the steepness and the curvature of the yield curve) are sufficient to explain almost all the variations in interest rates. Obviously, this is an encouraging result, because fixed income risk management can be reduced to managing the effects of these three risk factors on the portfolio value, regardless of the number and characteristics of the bonds included in the portfolios.

This paper explores this ability of PCA to explain interest rate movements and bond returns and extends previous works in this area by analyzing two additional issues. First, the improvement over typical one- and two-factor term structure models; and second, the stability of the principal component model. To this end, we focus on a primary European market, the Spanish bond market, over the period January 1992 through December 1999.

The structure of this paper is as follows. Section 2 offers a description of the principal component model and the one- and two-factor models employed in the comparative analysis. Section 3 presents relevant information on the Spanish government debt market and the data used. Section 4 describes and discusses the results of the empirical study. Finally, conclusions are stated in Section 5.

2. Theoretical framework

The main assumption behind any model for interest rate changes is that unexpected market movements can be summarized by a limited number of risk factors. Assuming that unexpected interest rate changes are linearly related to these factors, it follows that:

$$\Delta r_t = \sum_{s=1}^{k} \alpha_{ts} \Delta x_s + \varepsilon_t \qquad \forall t \in (0, \Upsilon) \tag{1}$$

where r_t is the spot interest rate for term *t*, α_{ts} are the coefficients that link interest rate changes to the systematic risk factors x_s, $s = 1, \ldots, k$, ε_t are error terms that summarize idiosyncratic risks, and $(0, \Upsilon)$ refers to the term interval.

In this context, the first task is to choose the variables that can be used as proxies of the unknown systematic risk factors on which the term structure of interest rates (TSIR in advance) is assumed to depend. This is an important issue, because the model's ability to explain TSIR movements will depend crucially on this choice.

Typical interest rate and principal component models offer different answers to this matter. The former usually opt for choosing the change in one or more spot rates as the proxies of the unknown factors, while the latter highlight the usefulness of PCA for extracting the best proxies from the history of TSIR movements.

When dealing with typical interest rate models, a common practice is to specify the set of spot rates that act as proxies on an ad hoc basis, although some empirical selection criteria may also be found in the financial literature. One of these, and one which we will employ in our empirical analysis, is the selection criterion proposed by Elton et al. (1990).

According to Elton et al., the set of spot rates best chosen as optimal proxies of the unknown factors is the one that maximizes an objective function, defined as:

$$\sum_{t} w_t R_{tx}^2 \operatorname{var}\left(\Delta r_t\right) \tag{2}$$

where $\operatorname{var}\left(\Delta r_t\right)$ is the variance of the changes in the interest rate for term *t*; R_{tx}^2 is the centered *R*-squared of the OLS regression of Δr_t on a constant *c* and the changes in the spot rates chosen as tentative proxies *x*; and $\{w\}_t$ defines the weighting scheme used to average across maturities.

In a single factor model, we are led to perform univariate regressions of each series of spot rate changes on each other and retain the products $R_{tx}^2 \operatorname{var}\left(\Delta r_t\right)$ for all *t* and *x*; the optimal proxy will be the spot rate *x* with maximum performance across maturities according to (2). In a two-factor model, the proxies are the two spot rates that maximize the value of the objective function and thus regressions on each combination of two spot rates have to be carried out.

The key role of $R_{tx}^2 \operatorname{var}\left(\Delta r_t\right)$ in the objective function is explained by the fact that R_{tx}^2 can be expressed as:

$$R_{tx} = 1 - \frac{\operatorname{var}(\varepsilon_t)}{\operatorname{var}(\Delta r_t)} \tag{3}$$

and accordingly:

$$R_{tx} \operatorname{var}\left(\Delta r_t\right) = \operatorname{var}\left(\Delta r_t\right) - \operatorname{var}\left(\varepsilon_t\right) \tag{4}$$

Thus, for the explanation of each interest rate, the sets of proxies are ranked by their capacity to minimize the squared forecast error. The importance given to each interest rate in the choice of the optimal set of spot rates is determined by the weighting scheme, which should be related with the model's purpose[1].

Another point of Elton et al's methodology worth highlighting is the dimensionality (i.e., the number of spot rates that will be chosen as proxies of the unknown factors). In practice, this number is usually limited to a maximum of two due to the high number of regressions that must be made to determine the optimal set of proxies when the initial set of spot rates is sufficiently large to cover the entire TSIR.

Also, note that in Elton et al's framework the proxies of the unknown risk factors are limited to being particular spot rates (i.e., observable variables), an assumption that can be criticized if it is not lent weight by the data.

However, these shortcomings are overcome by principal component models, which offer a more flexible approach to dealing with the choice of risk factors. The key assumption underlying these models when applied to TSIR movements is that the multitude of variables that affect interest rates can be summarized by a few composite variables. These new variables are constructed by applying a principal component analysis (PCA) to past TSIR movements.

As a starting point, suppose that any TSIR shift can be characterized by a *n*-vector of spot rate changes with zero mean and covariance matrix Σ, that is:

$$TSIR\ shift = \left(\Delta r_1, \Delta r_2 \ldots, \Delta r_n\right) \tag{5}$$

PCA provides an alternative representation of TSIR shifts by using principal components instead of interest rate changes:

$$TSIR\ shift = \left(c_1, c_2 \ldots, c_n\right) \tag{6}$$

[1] For example, a long-term bond portfolio manager should give high weights to long-term rates at the expense of short-term rates. The contrary applies to a money manager.

Principal components (PC), however, are closely related to interest rate changes, since they are linear combinations of the latter:

$$c_j = \sum_{t=1}^{n} u_{jt} \Delta r_t \qquad j = 1, \ldots, n \tag{7}$$

where u_{jt} are called principal component coefficients. Using matrix notation, this can be also expressed as:

$$C = UR \tag{8}$$

where $C = \begin{bmatrix} c_1 & \ldots & c_n \end{bmatrix}^T$, U denotes an nxn matrix with elements u_{jt} and $R = \begin{bmatrix} \Delta r_1 & \ldots & \Delta r_n \end{bmatrix}^T$.

At this stage, there has still been no reduction in dimensionality, since the number of PCs that can theoretically be constructed is also n. However, not all the PCs are equally significant, because the first PC is chosen to explain the maximum percentage of the total variance of interest rate changes, the second is chosen to be linearly independent (i.e., orthogonal) from the first PC and explains the maximum percentage of the remaining variance, and so on. Consequently, if interest rate changes are highly correlated, it might be expected that only a few PCs would be necessary to capture TSIR movements. Moreover, these PCs are purposely constructed to be independent, which makes interest rate risk management a simpler task because each risk factor can be treated separately.

In order to construct the PCs, PCA focuses on the covariance matrix of spot rate changes. Since this matrix is symmetric, by using well-known results from matrix calculus we know that Σ has n normalized and linearly independent eigenvectors, $U_1, \ldots, U_n$, corresponding to n positive eigenvalues, $\lambda_1, \ldots, \lambda_n$. In fact, Σ can be factored as:

$$\Sigma = U^T \Lambda U \tag{9}$$

where $U = \begin{bmatrix} U_1 & \ldots & U_n \end{bmatrix}^T$ and Λ is a diagonal matrix with elements $\lambda_1, \ldots, \lambda_n$ along the diagonal.

As is usual, the n eigenvalues are obtained by resolving the equation:

$$\det\left[\Sigma - \lambda I\right] = 0 \tag{10}$$

where I is the identity matrix.

And the eigenvectors are the normalized solutions to the set of equations:

$$\left(\Sigma - I\lambda_j\right)U_j = 0 \qquad j = 1, 2, \ldots, n \tag{11}$$

The following step in the PCA consists of ranking the eigenvectors and eigenvalues in order of magnitude of the latter. In this way, the vector of the principal component coefficients of the first PC is given by the eigenvector corresponding to the highest eigenvalue, its variance being the magnitude of this eigenvalue; the vector of the coefficients of the second PC is given by the eigenvector with the following eigenvalue, and so on.

Since any TSIR can be described by the full set of PCs and the variance of each PC is given by the magnitude of its eigenvalue, the total variance of the set of interest rate changes is:

$$\sum_{j=1}^{n} \lambda_j \tag{12}$$

and the proportion of this variance explained by the jth PC is:

$$\frac{\lambda_j}{\sum_{j=1}^{n} \lambda_j} \tag{13}$$

In order to obtain a model for interest rates changes, we simply reverse equation (8):

$$R = U^{-1}C \tag{14}$$

Bearing in mind that eigenvectors are linearly independent and are also normalized we obtain:

$$R = U^T C \tag{15}$$

or equivalently:

$$\Delta r_t = \sum_{j=1}^{n} u_{jt} c_j \qquad t = 1, \ldots, n \tag{16}$$

Dimensionality is then reduced by disregarding those PCs that are of minor importance to explain interest rate changes (i.e., those with the lowest eigenvalues), not only because we are looking for a parsimonious model but also because we might thereby achieve some kind of noise reduction since the data not contained in the first PCs may be partly or mostly due to noise.

Assuming that we retain the first *k* PCs, expression (16) can be rewritten as:

$$\Delta r_t = \sum_{j=1}^{k} u_{jt} c_j + \varepsilon_t \qquad t = 1, \ldots, n \tag{17}$$

where ε_t is an error term that summarizes the changes not explained by the first *k* PCs.

It is worth pointing out that the above expression coincides with the final equation of a factor analysis in which the method employed to extract common factors is PCA. However, one thing needs to be clarified: when PCs are standardized, as is usually done in factor analysis, expression (17) has to be replaced by the following:

$$\Delta r_t = \sum_{j=1}^{k} \left(u_{jt} \sqrt{\lambda_j} \right) c_j^* + \varepsilon_t \qquad t = 1, \ldots, n \tag{18}$$

where $\mathrm{var}\left(C^*\right) = I$.

In this framework, the coefficients $u_{jt}\sqrt{\lambda_j}$, which measure the impact of each factor (principal component) on each original variable (interest rate changes), are called factor loadings. In order to simplify our notation, we rewrite expression (18) in terms of factors (*f*) and loadings (*l*), obtaining the final expression:

$$\Delta r_t = \sum_{j=1}^{k} l_{tj} f_j + \varepsilon_t \qquad t = 1, \ldots, n \tag{19}$$

Compared to typical interest rate models, principal component models do not require any kind of regression or historical fitting. Moreover, they do not assume that the variables that best explain TSIR movements are observable, since the choice is made by the data themselves. Litterman and Scheinkman (1991) illustrate this point with the following example: "[...] it is widely believed that changes in Federal Reserve policy are a major source of changes in the shape of the yield curve. If so – even though we have no clear idea of how to measure 'policy'- we can insulate ourselves against Fed policy changes provided only that we can determine the relative effects of these changes on the returns to bonds of different maturities". This is achieved, however, at the expense of model simplicity, since PCs are artificial variables that might be difficult to interpret.

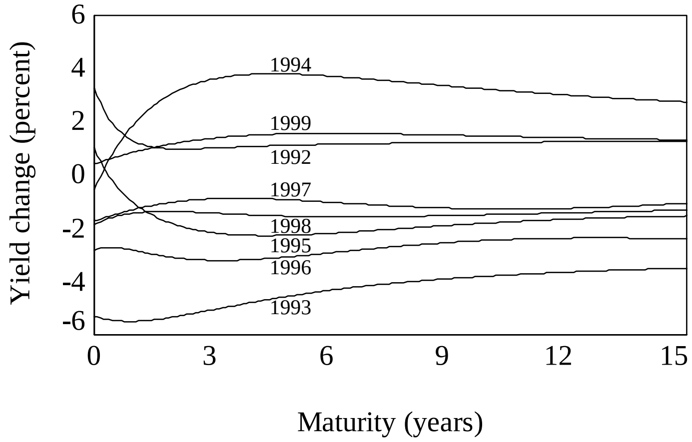
The rest of the paper, in which we carry out a detailed empirical analysis of the performance of both models, will be dedicated to weighing up the pros and cons of the respective approaches.

3. Data description

Our empirical analysis focuses on the Spanish government debt market, the fourth in importance in the European Union. The sample period extends from January 1992 to December 1999, providing eight years of daily data[2]. This constitutes a very valuable period for analyzing the ability of interest rate models to capture TSIR movements, since the Spanish TSIR changed dramatically during the nineties, as it did in other European countries. Figure 1, which shows the Spanish TSIR shifts each year from 1992 to 1999, illustrates this fact.

[2] We are restricted by the limited liquidity of the Spanish government debt market in the medium and long term prior to 1992.

Figure 1. Spanish TSIR shifts from 1992 to 1999



Note: Each curve corresponds to one year's shift. Zero-coupon rates have been extracted from coupon bond prices using the Nelson and Siegel (1987) methodology.

The origin of the Spanish TSIR movements is to be found both in international factors (the crisis of the European Monetary System form the last quarter of 1992 to the middle of 1993, the instability of the international bond markets during 1994 and the period of economic instability in the Euro area in 1999) and domestic factors (primarily, the convergence of Spanish interest rates with European levels).

All these forces resulted in fluctuations in the Spanish TSIR, which led to a net decrease in interest rates of 5.66 percent. The reduction was greater for the short end of the term structure (nearly 8 percent) and lower for the far end (around 4.5 percent), following the twist in the TSIR which took place during 1994.

The data used in this paper are the daily average prices of bonds traded in the Spanish government bond market and the daily average yields of repo transactions. The Banco de España, which manages the book-entry system of the market, publishes both data sets, from which we also extract zero-coupon bond yields using the methodology proposed by Nelson and Siegel (1987).

Since bond liquidity is not homogeneous either between bonds or with time, our empirical analysis focuses on a subset of the set of available bonds which fulfil the following two conditions:

(i) they were negotiated during at least ten days in each month of the six-month period and (ii) they represented a minimum of 2.5% of the six-monthly trading volume. This filter should ensure that bond returns are primarily affected by interest rate changes, and not by other factors like liquidity premiums or other idiosyncratic factors that go beyond the scope of this paper.

4. Performance and stability

Our empirical analysis has two well-defined parts. On the one hand, the interest rate models are estimated using historical spot rate changes. On the other, we evaluate the performance of the models by analyzing the ability of each to explain TSIR shifts and bond returns[3].

Weekly data are used for calculating both interest rate changes and bond returns over the observation period January 1992 through December 1999 in order to have available a large set of observations with significant changes and returns[4]. TSIR shifts are described by the series of unexpected interest rate changes for the series of maturities identified by RiskMetrics[5] for the Spanish money and bond markets. This set, which is similar to others employed in previous works[6], includes twelve unevenly spaced maturities, namely, one, three and six months and one, two, three, four, five, seven, nine, ten and fifteen years to maturity. The reason for using unexpected changes is twofold. First, this allows us to calculate bond returns as a sum of two components, the riskless return and the unexpected return due to unexpected changes in interest rates. And second, according

[3] Further evidence regarding the immunization performance of the optimal keyrate model, the principal component model and other competing models and portfolio designs is offered in Soto (2004) using the same data base employed in this paper.

[4] The use of weekly data reduces the day-of-the-week effect. We use Wednesday-to-Wednesday data because this reduces to a minimum the number of lost data. When lost data are found, we take prices and returns corresponding to the previous business day, which in all cases was Tuesday. We have eliminated two weeks of 1993 from the sample (May 12-May 19 and July 28–August 4) because of atypical data. These two weeks include the dates of the last ESP devaluation during the crisis of the European Monetary System and the enlargement of the bands of the System to ± 15%, respectively.

[5] See RiskMetrics (1996).

[6] For example, Litterman and Scheinkman (1991) use 6 months and 1, 2, 5, 8, 10, 14 and 18 years to maturity; Kahn and Gulrajani (1993) use 1, 2, 3, 4, 5, 7, 10 and 30 years to maturity; Bliss (1997) uses 3 and 6 months and 1, 2, 3, 5, 7, 10, 15 and 20 years to maturity.

to the financial literature, we should thus expect gains in model stability[7].

In order to deal with the stability analysis, we estimate the models using two different procedures. On the one hand, the models are estimated over the entire sample period 1992-1999. On the other hand, the models are estimated quarterly over successive 1-year windows, which fulfills the minimum requirements suggested by BIS to compute market risks (1-year samples and quarterly updating). In this way, conclusions concerning model stability can be drawn from a comparison of the sets of coefficients estimated and the risk factors chosen and the contrast between the errors made by each model in explaining TSIR shifts and bond returns.

4.1 Obtaining the models

As previously discussed, we consider two different interest rate models. The first one is a typical interest rate model, where risk factors correspond to specific spot rates. Similarly to Elton et al. (1990), we assume a univariate and a bivariate configuration[8], whereby the optimal set of one/two interest rates is obtained by applying the selection criterion discussed in section 2 with a flat weighting scheme. We name this model *optimal keyrate model* in order to distinguish it from the statistical factor model and point out that risk factors correspond to keyrates that are optimal according to the selection criterion of Elton et al.

The second model is the *principal component model*, which is constructed by applying a principal component analysis to the covariance matrix of interest rate changes.

The existence of a link between interest rate level and volatility makes it advisable to deal with interest rate changes expressed as growth rates (change in interest rate divided by initial interest rate)[9]. To do this, we modify the model specifications slightly. The optimal keyrate model

[7] In this regard, Babbel (1983) writes "application of the pure-expectation-shocks definition produced lower, more stable betas that were more easily predicted".

[8] See Elton et al. (1990) and Navarro and Nave (1997, 2001).

[9] This definition for interest rate changes is also usually employed in risk management systems. See, for example, RiskMetrics (1996), Dowd (1998) or Ahlstedt (1998).

for the univariate case becomes:

$$\Delta r_t / r_t = \alpha_{t1} \left(\Delta r_{opt_1} / r_{opt_1} \right) + \varepsilon_t \tag{20}$$

and for the bivariate case:

$$\Delta r_t / r_t = \alpha_{t1} \left(\Delta r_{opt_1} / r_{opt_1} \right) + \alpha_{t2} \left(\Delta r_{opt_2} / r_{opt_2} \right) + \varepsilon_t \tag{21}$$

where r_t is the spot interest rate for term *t*, α_{tj} are the coefficients that link interest rate changes to the optimal keyrate r_{opt_j} and ε_t are error terms that summarize idiosyncratic risks.

Regarding the principal component model we obtain:

$$\Delta r_t / r_t = \sum_{j=1}^{k} l_{tj} f_j + \varepsilon_t \tag{22}$$

where l_{tj} is the factor loading of the spot rate for term *t* on the factor (or principal component) f_j and *k* is the number of risk factors.

Both models are estimated using full-period data and successive four-quarter samples that overlap except in one quarter. Therefore, we obtain results for the overall period and for 29 subperiods of 1-year’s length.

Beginning with the univariate optimal factor model, Table 1 shows the number of subperiods in which each interest rate (in columns) is ranked in each position (in rows) according to the Elton et al. selection criterion. The 4-year, 5-year or 7-year rates (i.e., the intermediate maturity rates) are the clearest candidates to act as the optimal keyrate for the univariate model. It should be noted that interest rates for the shortest terms are the worst candidates, because this finding contrasts with the focus put on the development of short-term interest rate models by financial researchers.

Table 1. Interest rates' rankings for the univariate optimal keyrate model

Ranking	Interest rate											
	1-m	3-m	6-m	1-y	2-y	3-y	4-y	5-y	7-y	9-y	10-y	15-y
1		2	5				4	6	1			
2	1	5	1				6	5	2	1		1
3	3		1	3		6	1	6	8			
4	2			2	2	5	7	7	11	1		3
5	1		1		8	5	6	2	2	8	1	4
6				2	5	10	5		1	12	14	2
7				2	10	3		3	1	1	3	1
8		1		11	3				3		3	6
9		1	4	8	1					6	1	2
10		3	17	1							7	1
11	3	17										9
12	19											
Sum	29	29	29	29	29	29	29	29	29	29	29	29

Among these three rates, we select the *5-year interest rate* as the optimal keyrate for the overall subperiods. This rate is also optimal for the overall period 1992-1999. Once this selection has been made, the following step consists of obtaining the coefficients of expression (20) for each subperiod and the overall period. Figure 2 depicts the coefficient estimates for the subperiods that coincide with actual years (thin lines) and for the overall period (thick line), while Figure 3 shows the time series of coefficient estimates for the successive 1-year subperiods for a subset of six rates.

Figure 2. Interest rate sensitivities to the 5-year rate

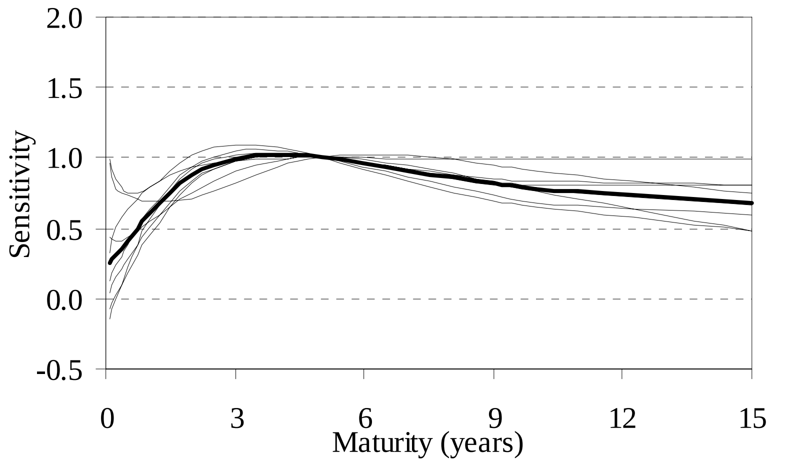
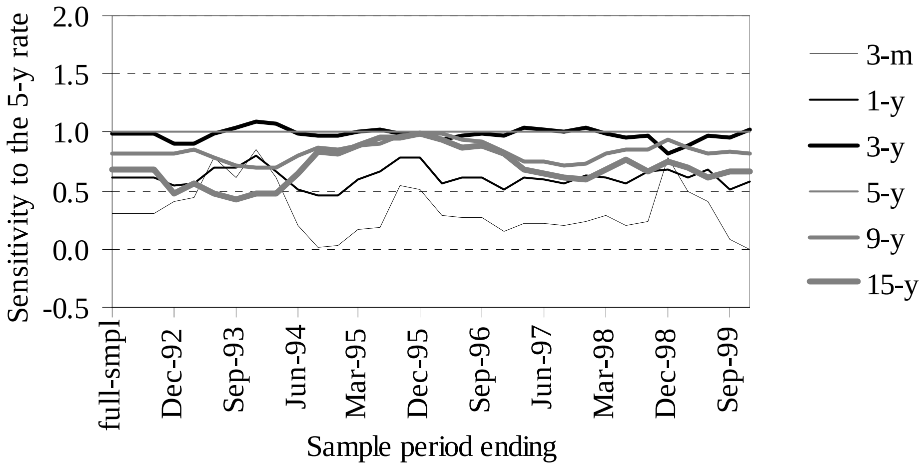


Figure 3. Time-series behavior of interest rate sensitivities to the 5-year rate



Note: The flat segment at the beginning of each series correspond to interest rate sensitivities to 5-year rate estimated over the full-period 1992-1999. These sensitivities are reported for comparative purposes.

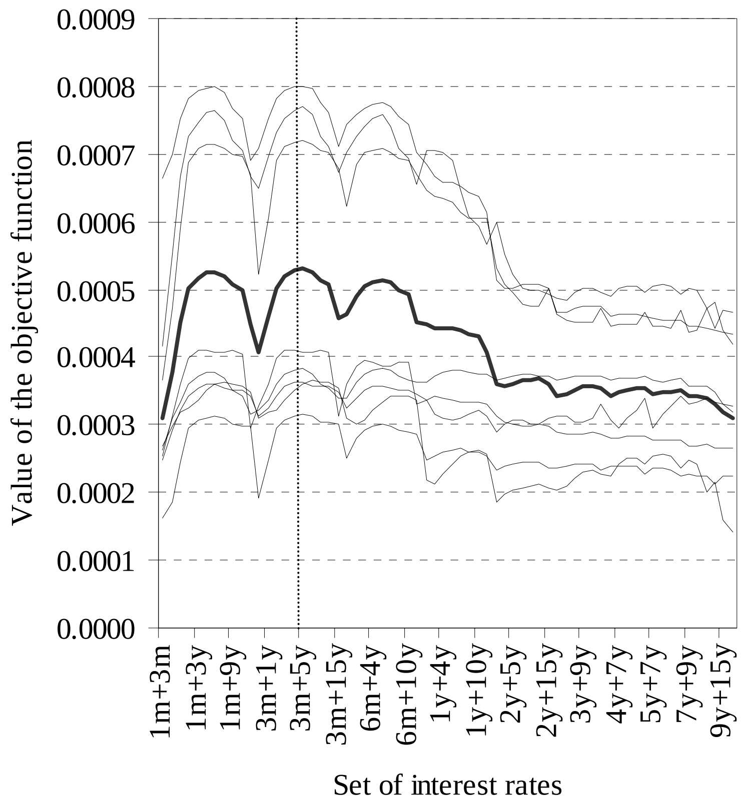
Figure 2 reveals that intermediate-term interest rates are closely related to the 5-year rate, but the nearer we get to shorter or longer maturities the link between interest rates and the 5-year rate diminishes, particularly for short-term rates. Figure 3 shows that the reduced relationship between the longest and shortest-term interest rates and the optimal keyrate leads to some instability in the coefficient estimates for these terms.

If this instability is due to a limited ability of the univariate optimal keyrate model to explain the movements of the overall TSIR, we would expect stability to increase in the case of the bivariate optimal keyrate model.

For the bivariate model we proceed as before, that is, we compute the rankings for all possible combinations of two interest rates. Although the rankings cannot be detailed as in Table 1, Figure 4 offers a general view of the pattern of the results, showing the values of the Elton et al. criterion for each combination of two interest rates for each of the subperiods that coincide with actual years (thin lines) and for the overall period (thick line). These values clearly reveal that the sets of interest rates that perform best combine a short-term interest rate (i.e., the 1 or 3-month rates) with an intermediate-term interest rate (i.e., the 4, 5 or 7-year rates). Again, this finding contrasts with the wide variety of two-factor interest rate models that select a short-term rate and a long-term rate as state variables.

Among the above sets of interest rates, we choose as optimal keyrates for the bivariate model the set formed by the *3-month rate* and the *5-year rate*, since this set of interest rates is optimal for 19 out of 29 subperiods and also for the overall period 1992-1999.

Figure 4. Values of the objective function for each set of two interest rates

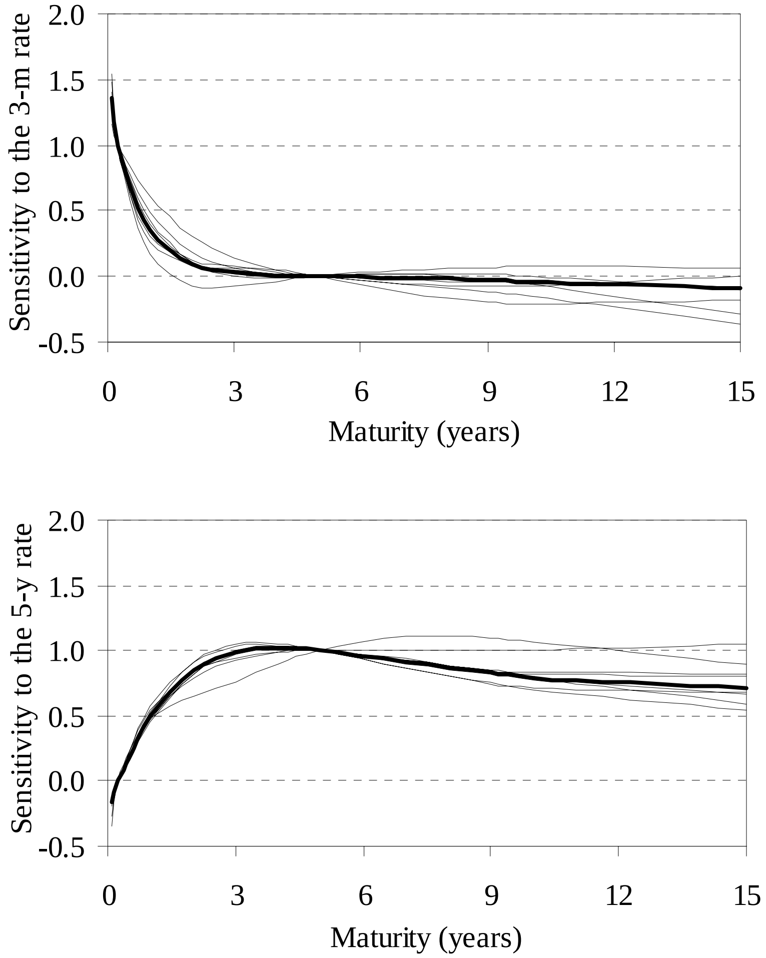


Note: Each set of interest rates is composed of two rates. The first one is the shorter-term rate. We start with the 1-month rate and add every single longer-term rate. We continue with the 3-month rate and add every single longer-term, and so on. For this reason, as the maturity of the shorter-term rate increases the number of combinations gets lower.

Figure 5 shows the coefficient estimates[10] for the subperiods that coincides with actual years (thin lines) and for the overall period (thick line), and Figure 6 show the time-series behavior of these coefficient estimates for the successive subperiods.

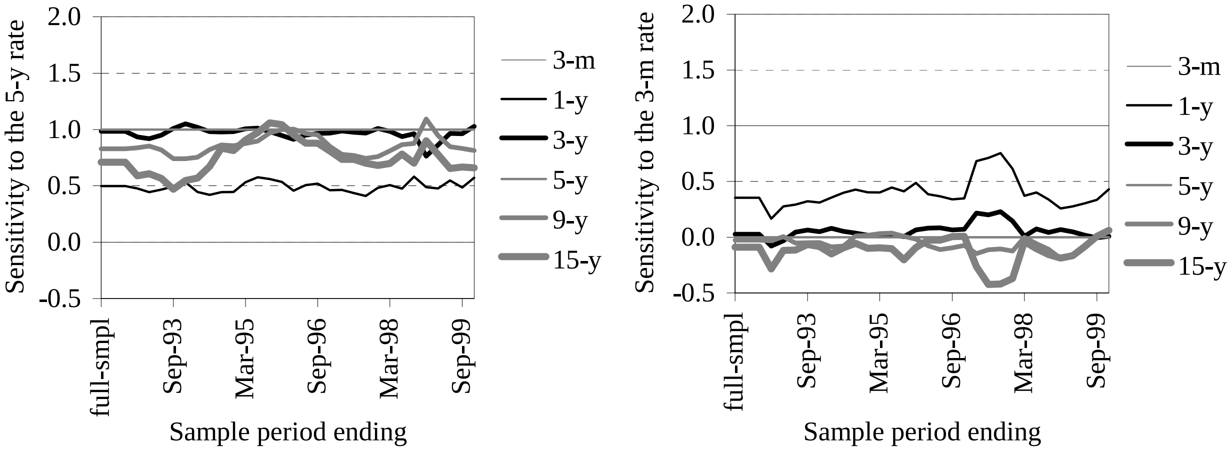
[10] In order to avoid collinearity problems, the regressions are performed on the 3-month rate and the difference between the 5-year rate and the 3-month rate. After obtaining the models, the original coefficients are reconstructed.

Figure 5. Interest rate sensitivities to the 3-month and 5-year rates



For the overall period, the coefficient estimates reveal that the sensitivity of interest rates to the 3-month rate falls rapidly to reach zero as maturity nears 5 years. Inversely, the sensitivity to the 5-year rate rapidly increases as we move from short to longer-term interest rates, reaching a maximum of nearly one for intermediate maturities. Quarterly estimations, however, reveal a significant instability in the sensitivity of short-term interest rates to the 3-month rate and of long-term interest rates to both the 3-month and 5-year rates. Comparing Figure 5 and Figure 6 with Figure 2 and 3, we can conclude that the instability registered by the sensitivities of short-term rates to the 5-year rate in the univariate model has been transferred to the sensitivities to the 3-month rate in the bivariate model, where it is less pronounced. Nevertheless, the instability of the sensitivities of long-term rates still remains. This might reveal that short-term rates are difficult to explain, even by specific short-term rates, and long-term rates cannot be described properly by the movements of the TSIR in the short and medium terms.

Figure 6. Time-series behavior of interest rate sensitivities to the 3-month and 5-year rates



Note: The flat segment at the beginning of each series correspond to interest rate sensitivities estimated over the full-period 1992-1999. These sensitivities are reported for comparative purposes.

Now we focus on the principal component model, where the risk factors (i.e., principal components) and their impact on the TSIR (i.e., factor loadings) are identified and specified by means of a principal component analysis (PCA) to the covariance matrix of interest rate changes. We also carry out an orthogonal rotation of the initial solution obtained from PCA in order to conform to the usual practice of interpreting the main movement of the TSIR as a parallel shift while maintaining the desired orthogonality of risk factors[11].

As before, we deal with the 29 subperiods of 1-year's length and the overall sample period 1992-1999. The results are summarized in Table 2 and Figure 7. Table 2 reports the explanatory power of the first three factors in the subperiods that coincide with actual years and in the overall period. Figure 7 shows the factor loadings for the same subperiods (thin lines) and for the overall period (thick line).

[11] Bliss (1997) offers a detailed explanation of the procedure.

Table 2. Explanatory power of the risk factors (percent)

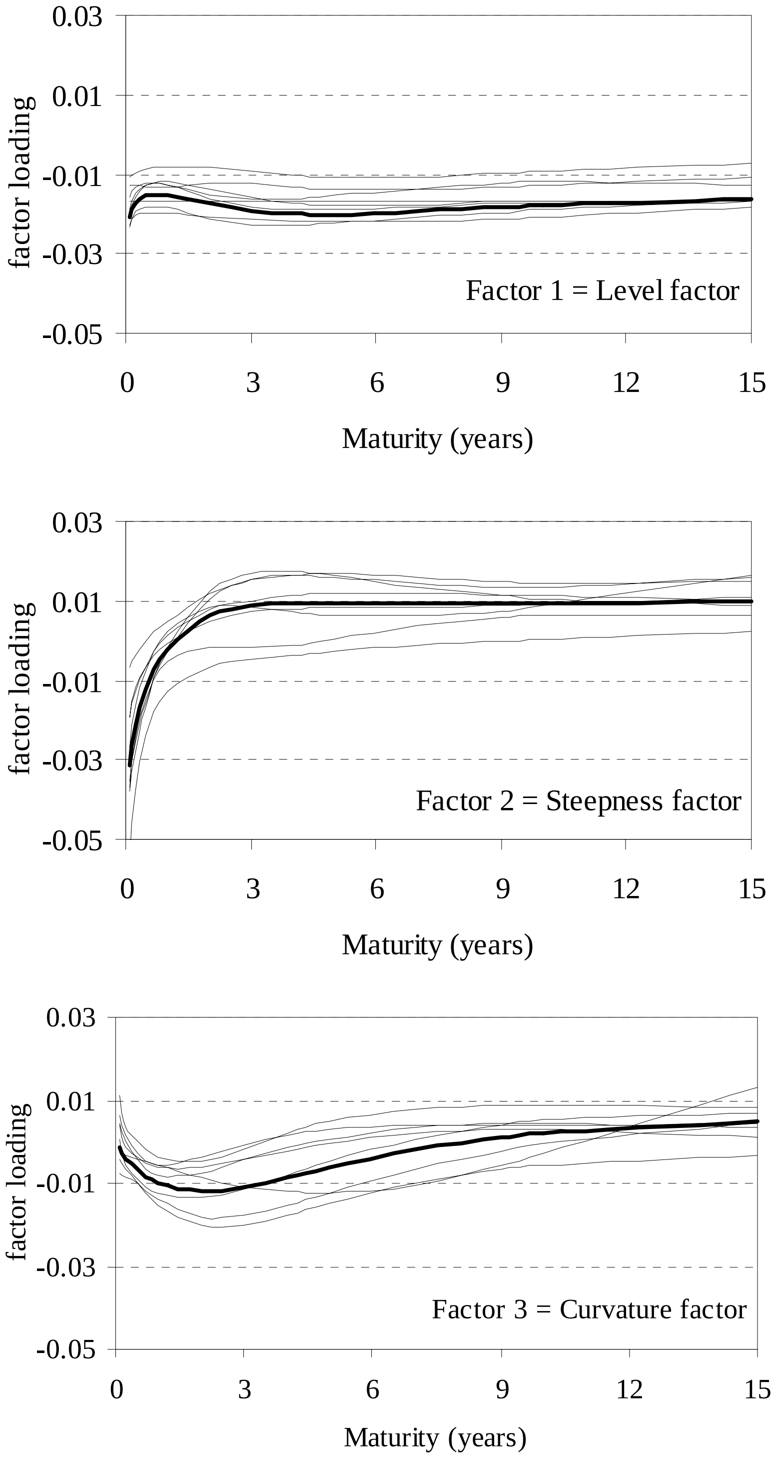
Sample	Factor 1	Factor 2	Factor 3	Sum
1992-1999	56.956	32.238	8.037	97.231
1992	52.553	29.129	14.930	96.611
1993	33.238	50.840	14.516	98.594
1994	54.481	41.232	2.558	98.271
1995	71.742	23.565	3.125	98.433
1996	56.742	36.093	4.279	97.114
1997	22.038	38.250	38.701	98.989
1998	40.032	46.978	9.810	96.820
1999	51.953	39.627	7.446	99.026

Note: Factor 1 is the first PC obtained over the full-sample period, Factor 2 is the second PC and Factor 3 is the third PC. Therefore, for the full-sample period the factors are in order of importance. For the 1-year samples, Factor 1, 2 and 3 correspond to the factors that affect TSIR in a way similar to how Factor 1, 2 and 3 affect the TSIR over the full-sample period. Consequenlty, for these samples the factors are not necessary in order of importance.

Our results are similar to previous studies. Three risk factors explain almost the entire variance of interest rate changes in both the overall period and the one-year samples. The first factor basically represents a parallel change in TSIR, which is why it is usually named *level factor*. The main effect of the second factor is to modify the slope of the short end of the term structure, for which reason it is named *steepness factor*. The third factor, or *curvature factor*, effects the curvature of the TSIR over short and medium terms.

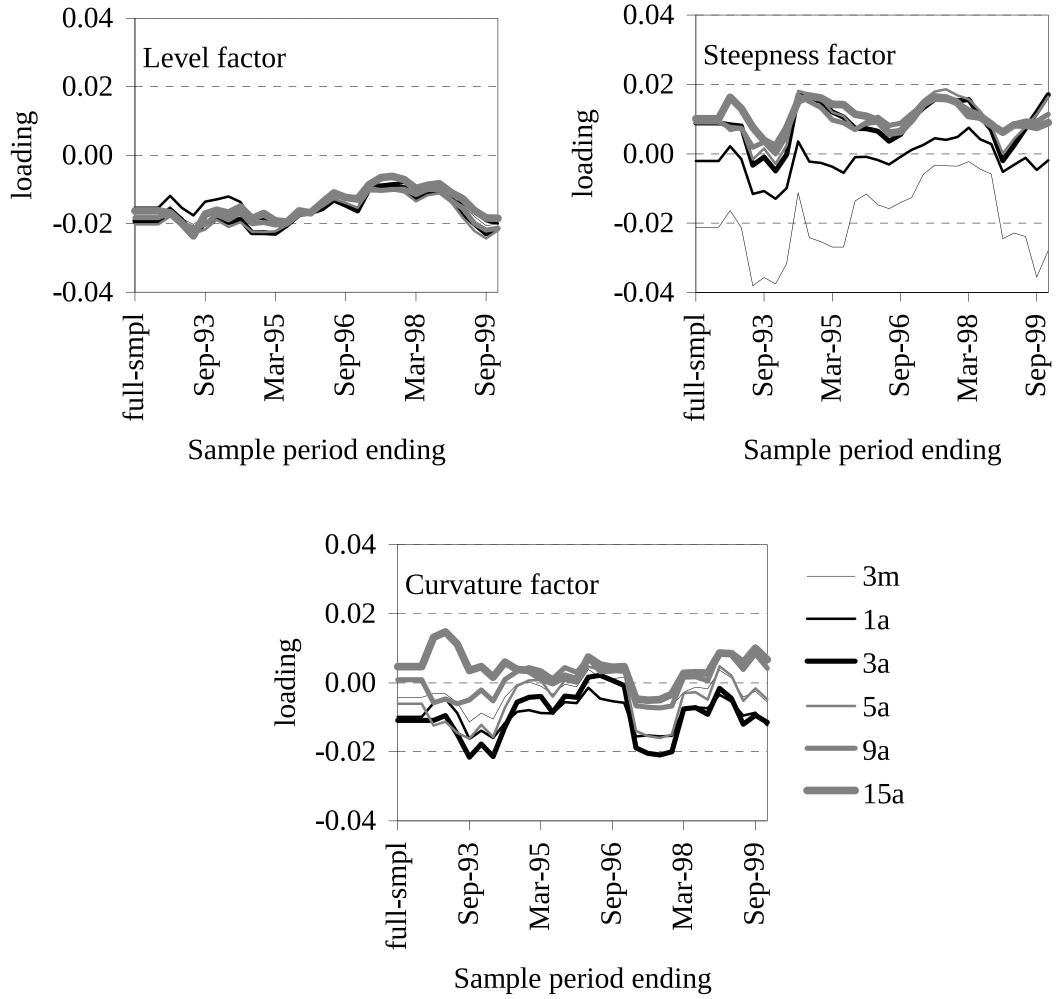
Apart from these general descriptions, the exact influence of the three risk factors, and even their importance in explaining the behavior of TSIR, depends crucially on the sample period. For example, Table 2 shows that in 1993, 1997 and 1998 the most important factor was not the level factor; moreover, in 1997 the ranking of the factors is the opposite of what was to be expected. Taking in account that our factors have a unit variance per se, this fact emphasizes a substantial variability in factor loadings. This can be confirmed by the inspection of Figure 7 and also of Figure 8, which reveals wide divergences in the loadings on the steepness and curvature factors primarily. The lack of constancy in the covariance matrix of interest rate changes underlies the variability of factor loadings, since the principal component model fully depends on this matrix as the only source of information.

Figure 7. Impact of each factor on the TSIR (factor loadings)



Note: Since factors are standardized, factor loadings indicate the change in TSIR resulting from a 1-standard-deviation shift in each factor.

Figure 8. Time-series behavior of factor loadings



Note: The flat segment at the beginning of each series correspond to factor loadings obtained for the full-period 1992-1999. These loadings are reported for comparative purposes.

4.2 Explaining TSIR shifts and bond returns

In this section we analyze the ability of the optimal keyrate model and the principal component model to explain weekly TSIR movements and bond returns over the period January 1992 through December 1999. Our aim is twofold. On the one hand, we are concerned with the comparative analysis between the optimal keyrate model and the principal component model, and on the other, we attempt to evaluate whether the signs of instability obtained in the previous subsection affect the performance of the models.

The measure we use to assess the performance of each model is the absolute deviation between the estimated return and the effective return. These deviations are also divided by the effective return in order to obtain comparable data for different bonds or interest rates and different volatility contexts. According to this, in the case of the TSIR fitting, for each week, interest rate and model we calculate:

$$Abs\left[\frac{\Delta r_t^{est} - \Delta r_t^{eff}}{\Delta r_t^{eff}}\right]\cdot 100 \tag{23}$$

where Δr_t^{eff} is the effective unexpected change in spot rate for term *t* and Δr_t^{est} is the estimated change.

When dealing with bond returns, for each week, bond and model we obtain:

$$Abs\left[\frac{R_i^{est} - R_i^{eff}}{R_i^{eff}}\right]\cdot 100 \tag{24}$$

where R_i^{eff} is the effective return of the bond *i*, and R_i^{est} is the estimated return.

Since the optimal keyrate model and the principal component model estimated over the entire sample period 1992-1999 are evaluated using the same sample period, our series of absolute deviations are series of in-sample errors. To obtain similar in-sample errors for the models estimated over the successive 1-year windows, we take the model estimated over the subperiod that ends in the quarter which the bond return or the interest rate change refers to (for example, the estimated interest rate changes and bond returns for the weeks of October, November and December 1998 are calculated from the models estimated over the period December 1997 to December 1998).

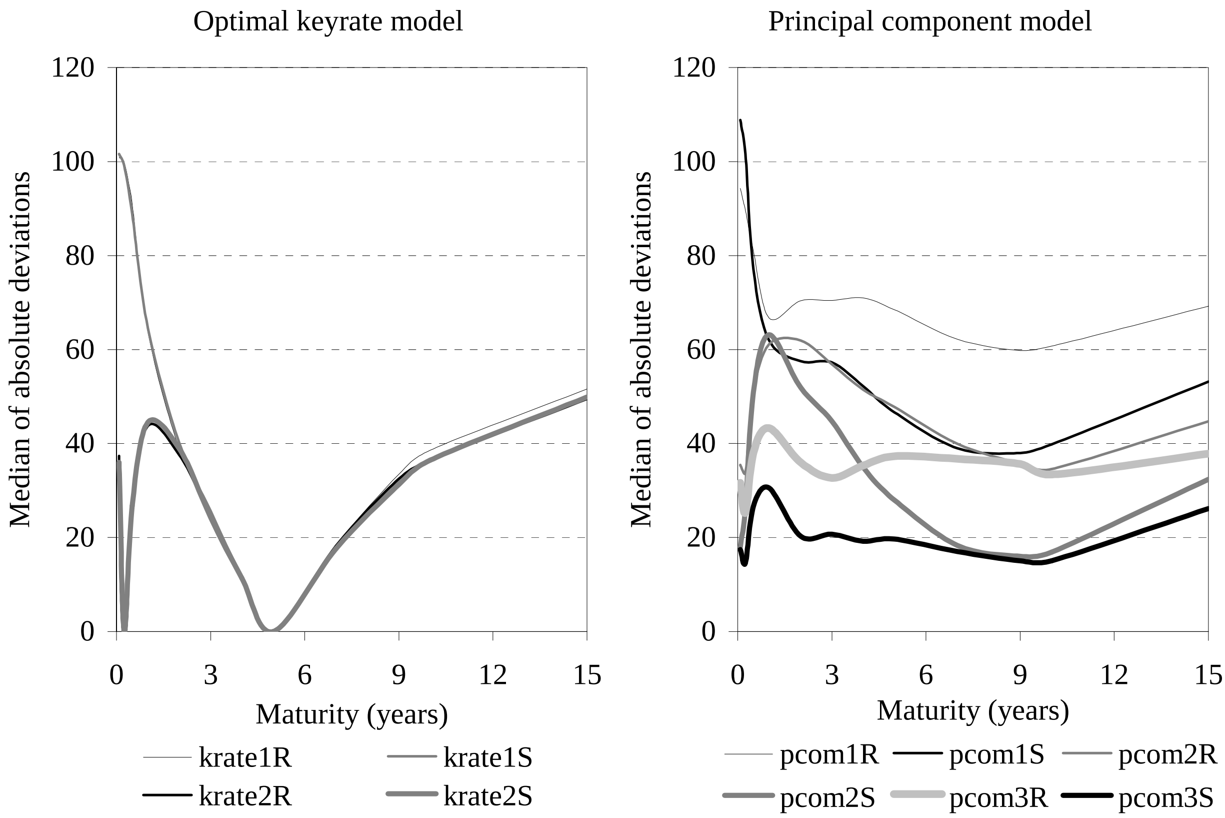
The number of risk factors for the optimal keyrate model may be one (the 5-year rate) or two (the 3-month and the 5-year rates) and the number of risk factors for the principal component model may be one (the level factor), two (the level and the steepness factors) or three (the level, steepness and curvature factors).

As a result of the two estimation procedures and the number of risk factors that can be considered, we have ten different approaches for explaining interest rate changes and bond returns. The names given to these approaches result from linking the abbreviation for the model ("krate" for the optimal keyrate model and "pcom" for the principal component model), the number of risk factors (1 or 2 for the optimal keyrate model and 1, 2 or 3 for the principal component model) and a code which refers to the estimation procedure (S corresponds to the single estimation for the overall period and R to the set of rolling estimations with a 1-year window). For example, pcom2R refers to the principal component model with a level factor and a steepness factor obtained from rolling estimations. Similarly, krate1S refers to the univariate optimal keyrate model estimated for the overall sample.

We first analyze the ability of each approach to explain interest rate changes. Figure 9 shows the median of the absolute deviations of estimated changes from actual changes for the four different specifications of the optimal keyrate model and the six specifications of the principal component model.

As regards the optimal keyrate model, the graph on the left of Figure 9 shows that the models with rolling estimations (R) perform similarly to the models estimated from full-period data (S). The pattern of deviations between estimated and effective interest rate changes is seen to depend on the number and location of the optimal keyrate/s. On this matter, the optimal keyrate model with a single risk factor (i.e., the 5-year rate) is unable to explain short-term interest rates and, as we move from the 5-year rate to longer-term rates, deviations gradually increase. The bivariate model (krate2) only differs from the univariate model (krate1) in its much higher ability to explain short-term interest rate changes. This is consistent with the fact that the second risk factor is a short-term rate (i.e., the 3-month rate), which adds information about the movements of the TSIR in the short end, but not about intermediate and long-term interest rates.

Figure 9. Absolute deviations between the effective interest rate changes
and the changes explained by each model

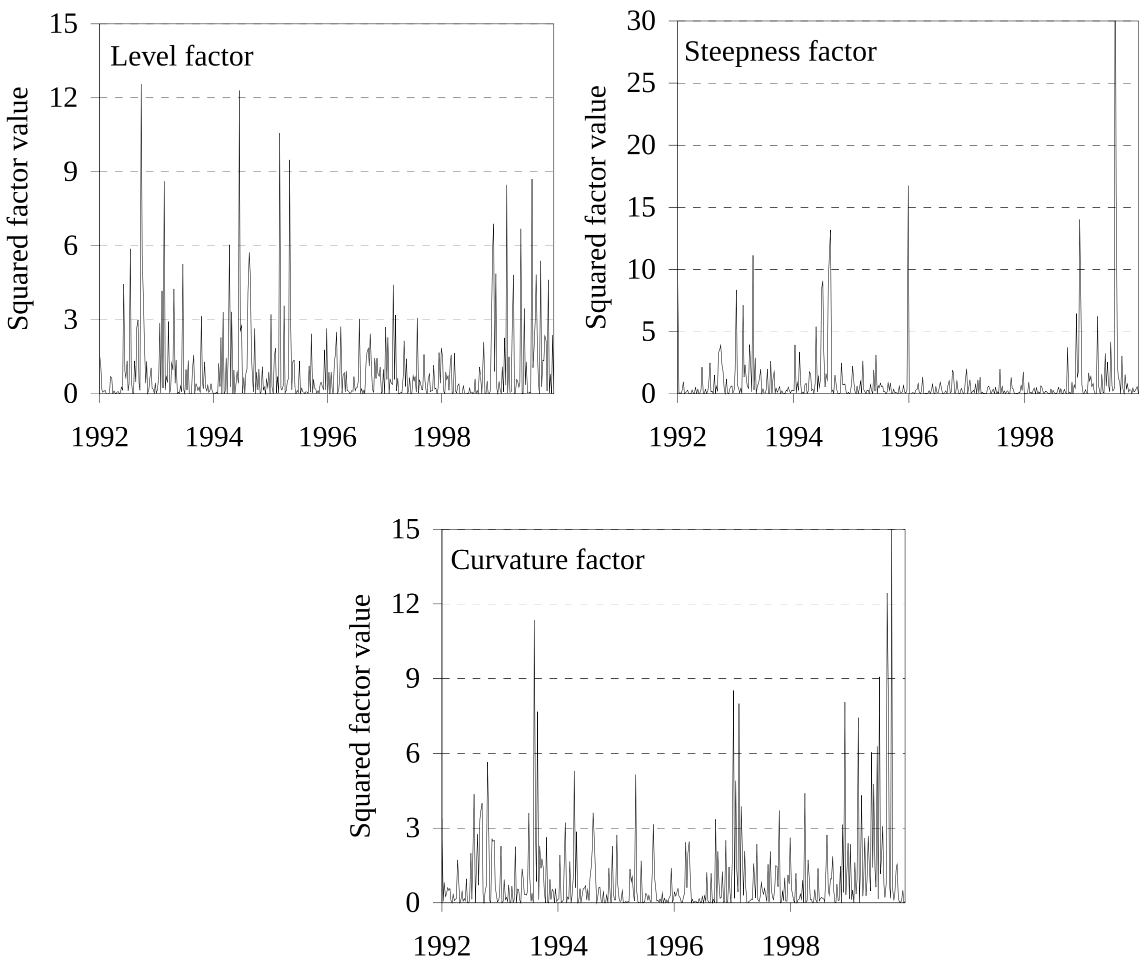


The graph on the right of Figure 9 corresponds to the six principal component models and offers quite different results.

The most outstanding result is that the models with rolling estimations perform much worse than the models estimated from full-period data, whatever the number of risk factors. This is especially true for medium and long-term interest rates. Indeed, the graph shows that pcom2S performs even better than pcom3R for the medium and long terms (i.e., four years and above). This is a worrying result, especially taking into account that our deviations are in-sample errors and therefore they are not expected to be higher for the models estimated from shorter samples.

This worsening performance is a result of the sharp signs of instability detected previously and indicates that the kind of movements registered by the TSIR cannot be identified properly from the most recent TSIR shifts. Unlike the optimal keyrate model, the principal component model is shown to need long periods for this task. This does not mean, however, that the most recent TSIR changes do not offer valuable information about what will happen in the near future. In fact, as is shown in Figure 10, the volatility of each risk factor shows a time-dependent path that can be exploited to obtain forecasts of the variance of the level, steepness and curvature factors[12].

Figure 10. Squared factor values



Note: Factor values correspond to the model estimated over the full-sample period 1992-1999.

These changing volatilities of the risk factors are a result of the changing variances and covariances of interest rates, since the factors are linear combinations of the initial interest rates

[12] Alexander and Chibumba (1998) first applied this approach consisting of combining the principal component analysis (or the statistical factor analysis) with a volatility model for each risk factor to Value at Risk calculations in portfolios including bond, exchange rate and stock positions. They assume a GARCH(1,1) specification for the volatility of the risk factors, for which reason they name the model Orthogonal-GARCH. Engel and Gizycki (1999) and Byström (2000) also offer evidence in favor of the Orthogonal-GARCH model.

changes. Note that this contrasts sharply with the fact that PCA is a static technique, unable to deal with the time-series behavior of interest rates. It is for this reason that the TSIR shifts identified from the 1-year samples have been revealed to be of limited usefulness. Even the shifts identified from the overall sample period are inefficient, since their extraction procedure does not recognize the existence of volatility dynamics. These arguments point to the suitability of dynamic factor models that combine a parsimonious factor structure with stochastic volatility.

Returning to our (static) principal component model, we again focus on the graph on the right of Figure 9 to show that deviations of estimated changes from effective interest rate changes diminish as risk factors are added in the model. In this regard, the inclusion of the steepness factor does not significantly improve the fitting of interest rates around 1-year, but it does so for both shorter and longer-term interest rates. The inclusion of the curvature factor reduces the errors made in the fitting of intermediate-term interest rates. This pattern of improvements is consistent with the series of factor loadings of Figure 7.

A comparison of the results of the optimal keyrate model with a single risk factor and the principal component model of equal dimension estimated from full-period data enables us to affirm that the former performs significantly better. For long and short-term interest rates, the principal component model and the optimal keyrate model offer the same results, but intermediate-term interest rates are better explained by the optimal keyrate model. Therefore, when dealing with univariate models, a traditional interest rate model seems to be the best option.

When two risk factors are considered, we obtain mixed results. The optimal keyrate model overcomes the principal component model estimated from full-period data for short and intermediate-term interest rates, but for long-term interest rates the principal component model offers the lowest errors.

Lastly, the principal component model with three risk factors estimated from full-period data is shown to offer a better explanation of TSIR shifts than the two-factor optimal keyrate model, except for interest rates close to the 3-month and 5-year interest rates. Moreover, unlike the optimal keyrate model, the performance of the principal component model is very similar across terms, which is clearly a desirable feature. Both arguments support the use of the principal component model over the typical two-factor interest rate models, provided that TSIR shifts are correctly identified from a long series of historical data.

We now analyze the consistency of these results using real bond data instead of fitted term structures. To this end, we obtain the weekly returns of the most liquid bonds of the Spanish bond market during the period January 1992 to December 1999[13] and compute the deviations between these returns and the returns estimated from each of our ten approaches. Since the above analysis has revealed that the fitting of interest rate changes differs across terms, we have thought it advisable to split the full sample of returns (2288 observations) into three sub-samples according to bond's maturities. The first sub-sample includes the returns of bonds with maturities up to three years (535 observations)[14]; the second sub-sample refers to bonds with maturities above 3 years and up to 7 years (921 observations); the third sub-sample collects the returns of bonds with maturities above 7 years (832 observations).

Table 3 reports the median of the absolute deviations of effective returns from estimated returns and the correlation coefficients between both sets of returns for the overall sample and the three sub-samples.

[13] For each week, we exclude those bonds with coupon payments and/or with a return lower than 25 basis points.

[14] The lower number of observations in the first sub-sample is due to the limited liquidity of the bonds with shortest maturities.

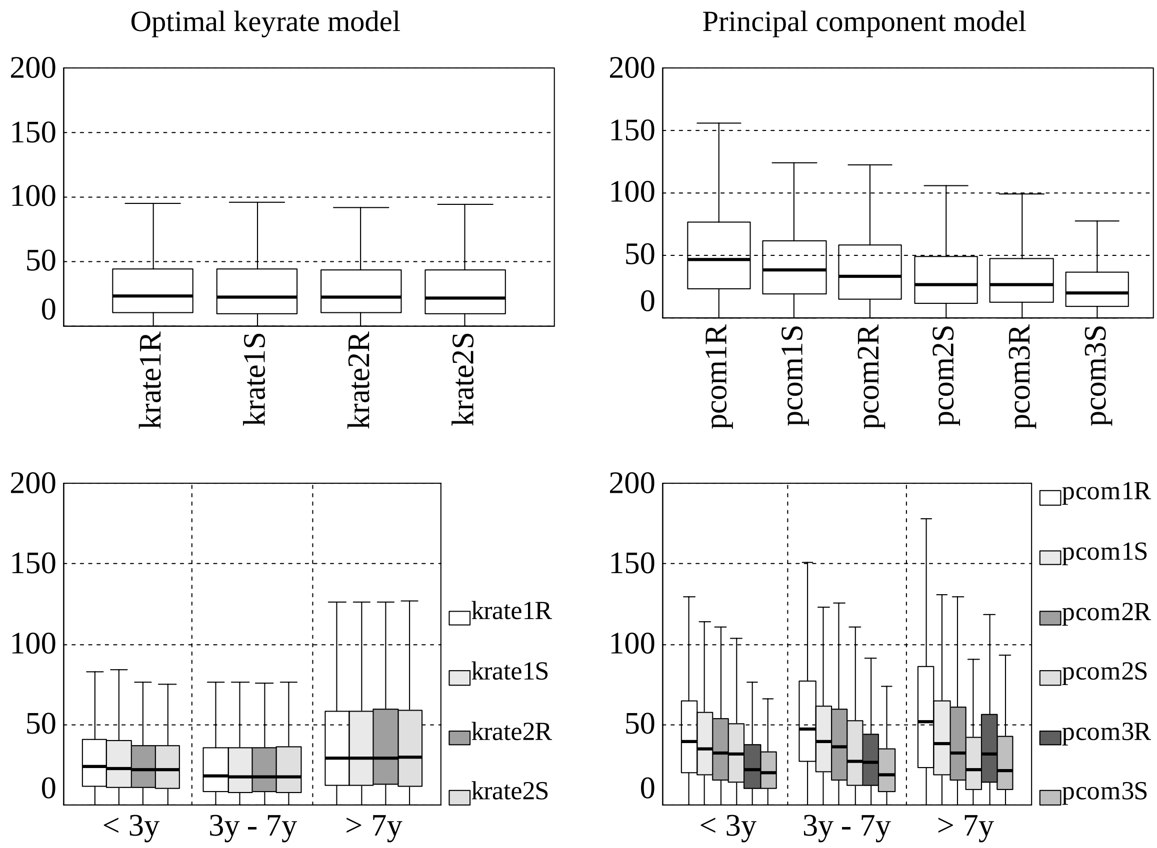
Table 3. Fitting of bond returns

Approach	Median of the absolute deviations of effective returns from estimated returns				Correlation coefficient between effective returns and estimated returns			
	Short maturities	Intermediate maturities	Long maturities	Full sample	Short maturities	Intermediate maturities	Long maturities	Full sample
krate1R	24.124	18.210	29.083	23.057	0.931	0.964	0.937	0.945
krate1S	22.965	17.688	29.397	22.699	0.927	0.963	0.937	0.945
krate2R	22.001	17.721	28.926	22.289	0.937	0.964	0.938	0.946
krate2S	21.808	17.416	29.808	22.176	0.937	0.964	0.938	0.946
pcom1R	39.660	47.542	51.672	46.931	0.784	0.789	0.816	0.805
pcom1S	35.132	39.273	38.462	37.963	0.863	0.856	0.878	0.870
pcom2R	32.367	35.984	32.531	33.642	0.861	0.871	0.921	0.901
pcom2S	31.457	27.387	21.738	26.398	0.892	0.919	0.959	0.941
pcom3R	22.288	26.497	31.651	26.850	0.943	0.951	0.944	0.946
pcom3S	19.902	19.092	21.218	20.002	0.955	0.963	0.960	0.960

A more detailed information about deviations is offered in Figure 11, which includes a set of two boxplots for each model: the upper graphs show the results for the overall sample of returns and the lower graphs the results for each of the three sub-samples of returns.

For the optimal keyrate model, Table 3 and Figure 11 show once more that the models with quarterly estimations perform similarly to the models based on full-period data, particularly in the case of the two-factor models. The improvements of the two-factor models over the single-factor models are significant in the case of bonds with maturities up to three years, which are the most affected by the dynamics of short-term rates, but nearly vanish for bonds with longer maturities. Finally, it can be seen that the pattern of the errors made when explaining TSIR shifts has been transferred to the fitting of bond returns. The best fitting is obtained for the returns of the bonds with intermediate maturities at the expense of shorter maturity bonds and, especially, longer maturity bonds.

Figure 11. Boxplots of the absolute deviations of effective returns from estimated returns



Note: In these boxplots (or box-and-whisker plots) each box is outlined by three horizontal lines, which mark the values of the third quartile, the second quartile (the median) and the first quartile. The vertical lines (or whiskers) extend upwards and downwards to a limit that represents 1.5 times the inter-quartile range. As a result, these graphs provide valuable information about the level, variability and asymmetry of the data.

The results for the principal component model for the overall sample and the three different sub-samples confirm the assertion that the models estimated over the overall period 1992-1999 perform significantly better than the models with rolling estimations. The above mentioned advantage of pcom2S over pcom3R in explaining long-term interest rate changes would explain why the deviations of effective returns from estimated returns for the set of bonds with maturities longer than 7 years obtained from pcom2S are lower than those obtained from pcom3R. Unlike the optimal keyrate model, the inclusion of additional factors in the principal component model is shown to be of clear interest for explaining the returns of each and every set of bonds more accurately.

The parallelism of these results with those obtained previously leads us to similar conclusions. Typical interest rate models where factors are identified with one or two specific interest rates are able to perform even better than the principal component model with up to two risk factors. However, the performance of both models when one or two risk factors are considered is shown to be significantly different across terms and maturities. The inclusion of a third risk factor in the principal component model overcomes this problem and leads to improvements over typical interest rate models, provided that TSIR shifts have been correctly identified from a long series of historical shifts.

5. Conclusions

In this paper, we deal with the Spanish bond market in an attempt to evaluate the performance of a kind of interest rate model that relies on principal component analysis to extract risk factors. We analyze the ability of this model to explain both interest rate changes and bond returns and compare it with the results of typical one and two-factor interest rate models.

Our empirical analysis reveals that the movements in the Spanish term structure can be summarized by the usual three principal components related to the level, steepness and curvature of the term structure. The principal component model with these three factors is able to offer a balanced explanation of interest rate shocks and bond returns across terms and maturities. In contrast, the one-factor interest rate model fails to capture the dynamics of both short and long interest rates and the two-factor interest rate model that of long interest rates.

As regards stability, our results reveal some variations in time in the principal component model that cannot be dismissed. In this sense, we highlight that this model’s performance deteriorates significantly when the model is estimated from the most recent data as opposed to using long-sample data. This fact is problematic from a practical perspective and points to the need to direct research toward parsimonious factor models with dynamic volatility structures.

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