Risk & Portfolio Management, Spring 2010
Homework 3

Risk Management and PCA. This exercise implements a general framework for evaluating the risk of an equity portfolio based on PCA and factor analysis. It is based on the model

$$R_s = \sigma_s \sum_{k=1}^{m} \beta_{sk} F_k + \sigma_s \left(\sqrt{1 - \sum_{k=1}^{m} \beta_{sk}^2} \right) G_s, \tag{1}$$

where R_s is the return of a given stock, σ_s represents its volatility, and the coefficients $\beta_{sk}, k = 1, ..., m$ represent the regression coefficients of standardized returns (R_s/σ_s) on the first m standardized PCA factors. We shall model the systematic factors F_k and the idyosyncratic factors G_s as Student t randowm variables with df=3.5 having mean zero and variance 1.

Part 1. Use Jan 29, 2010 as the reference date. Download historical data going back 1 year (252 days) up to the reference date. You can use the data posted for the previous assignment to save time. Perform a PCA on the returns and calculate the returns of the top 15 eigenportfolios, defined as

$$F_{kt} = \frac{1}{\sqrt{\lambda_k}} \sum_{i=1}^{500} \frac{V_i^{(k)}}{\sigma_i} R_{it}, t = 1, 2, 3, ..., 252. \tag{2}$$

Here λ_k is the k^{th} eigenvalue, $\left(V_i^{(k)}\right)_{i=1}^{500}$ is the k^{th} eigenvector, σ_i is the standard deviation of the i^{th} stock, and R_{it} is the return of stock i from the close of day t to the close of day $t+1$. Verify that these factors are "orthonormal" (uncorrelated and variance=1). Save the matrix F_{kt} in a file.

Part 2. Consider the sample portfolio of stocks and ETFs in the attached file (SAMPLEPORTFOLIO.csv), containing 75 positions, $Q_1, ..., Q_{75}$, on various stocks and ETFs.Download the corresponding price data for each ticker over the period 1/30/2009 to 1/29/2010 and calculate the matrix $\beta_{sk}, 1 \leq s \leq 75, 1 \leq k \leq 15$, associated with equation (1). Also compute the daily volatility of each stock (σ_s) based on the data.

Part 3. A *risk scenario* corresponds to a draw, or realization, of 75+15=90 independent standardized Student t random variables

$$F_1, ..., F_{15}, G_1, ..., G_{75},$$

with df=3.5 which will be used to simulate stock price changes according to the model in (1). Accordingly, the change in the value of the sample portfolio for a given risk scenario is

$$\Delta\Pi = \sum_{s=1}^{75} Q_s \sigma_s \left[\sum_{k=1}^{m} \beta_{sk} F_k + \left(\sqrt{1 - \sum_{k=1}^{m} \beta_{sk}^2} \right) G_s \right] \tag{3}$$

Compute the outcomes of 1000 risk scenarios for simulating the changes in value of the sample portfolio and sort them in decreasing order in terms of losses.

Part 4. Compare the 99% percentile loss with the corresponding value of Gaussian VaR ($\sigma \cdot 2.66$) and with the VaR at 99% level using Student-t with 3.5 degrees of freedom and the portfolio variance. Discuss the results.

Part 5. Some regulators suggest using an estimator for the variance of a stock which $max(\sigma_{1y}, \sigma_{3m})$, where σ_{1y} corresponds to the daily volatility computed using 1 year of daily data and σ_{3m} uses only the most recent 3 months of data. Using the same factors and Betas as before, compute the potential losses for the portfolio if you use this new estimator of volatility.