

PDE in Finance, Spring 2009: Homework 3
Prof. Avellaneda

1. Convertible Preferred Stock A convertible preferred stock (CPS) of a company is a security that pays interest ρ on a notional face amount (like a bond) and, in addition, may be converted into shares of stock of the company. The conversion ratio (R) is the amount of shares that can be exchanged for one CPS. For simplicity, we assume that the CPS pays a continuous (daily) interest ρ , *i.e.* that it pays the amount $\rho\Delta t$ over an interval of length Δt .

1. Derive a PDE for the price of a CPS as a function of the stock price, the volatility of the stock price, the reference interest rate r , the dividend d , and the conversion ratio R , assuming that the face value of the CPS is one dollar. Here is a suggested strategy:

a. Show that, formally, the relation between the value of the CPS at date $n\Delta t$ and date $(n+1)\Delta t$ is given by

$$V_n(S) = \rho\Delta t + \max(RS, E\{e^{-r\Delta t}V_{n+1}|S_n = S\}). \quad (1)$$

b. Deduce from (1) there exists an optimal conversion value S^* and that in the no-conversion region ($S < S^*$) the fair value of the CPS is $V(S)$, where

$$\rho + \frac{\sigma^2}{2}S^2\frac{\partial^2 V}{\partial S^2} + (r-d)S\frac{\partial V}{\partial S} - rV = 0. \quad (2)$$

c. Derive the general solution of the previous equation. Show that it has the form

$$V(S) = a_1 + bS^{\beta_1} + a_2S^{\beta_2}, \text{ where } \beta_1 > 1 \text{ and } \beta_2 < 0. \quad (3)$$

d. Show that only β_1 matters. Write down a formula for the fair value of the CPS as a function of σ, r, d, ρ and for the conversion value S^* .

2. How would the model change if the holder of the CPS is allowed to convert only if the stock price is above a strike price K ?

2. Concrete example. On January 9, 2008, Inverness Medical Innovations, Inc. (ticker symbol: IMAM) issued a convertible preferred stock (ticker symbol: IMAM-POB) to finance its purchase of Matria Pharmaceutical. The terms of the CPS were as follows:

- Face Value: \$ 400
- Coupon: 3% per annum
- Conversion Ratio: 5.577 shares per each CPS (compute R from this)
- Stock Price at inception: \$ 53.01

1. Due to the credit risk associated with this company, we discount the bond cashflows at a rate $r_{IMA} = r + s$, where s is the *credit spread* of IMA. In other words, we replace Equation (2) by

$$\rho + \frac{\sigma^2}{2} S^2 \frac{\partial^2 V}{\partial S^2} + (r - d) S \frac{\partial V}{\partial S} - r_{IMA} V = 0 \quad (4)$$

Derive the formula for the CPS assuming the modified rate due to credit risk.

2. Assume that $r = 1\%$, that IMA pays no dividend, and that the volatility of IMA was $\sigma = 60\%$ at inception. The convertible preferred began trading at \$ 230 USD. What is the implied credit spread?
3. The closing price on March 18, 2009 for IMA stock was \$ 26.08 and the preferred closed at \$159.09. Derive the value of the implied credit spread on this date (assuming the same volatility and interest rate of 1%).

3. American Options and dividends redux. Assume that S_t is a stock that pays a lump dividend of D dollars on the ex-dividend date T_1 and trades without the dividend thereafter. Consider an American call option on the stock with strike K and expiration $T > T_1$. Assuming a non-zero interest rate r and a volatility σ , compute

- (i) the fair value of the option assuming that a discrete dividend.
- (ii) the value of the option if we use an equivalent dividend yield from which gives the same forward price.
- (iii) Compare the two values. Is one necessarily larger than the other? Give examples with market data, where you take an American option on a dividend-paying stock and derive the implied volatility and the corresponding price using a continuous dividend yield.

4. Power Options Consider a contingent claim that pays the holder the amount

$$F(S) = \left(\frac{S}{S_0} \right)^\alpha$$

when the option expires, at time T . The constant α is a number different from zero (negative or positive). Price this contingent claim assuming that the asset satisfies a lognormal diffusion process. Calculate the hedge (Delta). Does the American-style version of this claim have early exercise premium?